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# THE OPTICAL EMISSION-LINE SPECTRUM OF CYGNUS A\*

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#### **ABSTRACT**

Spectrophotometric measurements are reported of the radio galaxy  $3C\ 405 = Cyg\ A$ , made using the imagetube image-dissector scanner on the Lick 120-inch (3 m) telescope. The measurements were reduced to energy units by comparison with scans of standard stars made with the same system on the same nights. The emission lines and continuum were measured in the spectral region  $\lambda\lambda 3346-6731$  (in the rest system of Cyg A). The interstellar extinction was determined from the measured Balmer-line ratios, assuming a Case B recombination spectrum and the standard Whitford reddening curve, and the measured line and continuum strengths were corrected for this extinction. The corrected line strengths are discussed for the information they contain on the physical conditions and the energy-input mechanism to the ionized gas. Photoionization by stars is ruled out by the great strength of [O I], [N I], and [S II]. Shock-wave heating is ruled out by the [O III] temperature, unless a large amount of ultraviolet ionizing radiation is emitted in the shock. Published calculations of photoionization by a synchrotron spectrum, extending far into the ultraviolet, approximately match the observed emission-line spectrum. Likewise, the observed Crab Nebula spectrum approximately matches Cyg A, except for abundance differences, in agreement with the idea that Cyg A is photoionized by a power-law spectrum.

Subject headings: radio sources — galaxies, individual — galactic nuclei — spectrophotometry

#### I. INTRODUCTION

Cygnus A=3C 405 is one of the brightest radio sources in the sky (Bolton and Stanley 1948), and was one of the first radio sources to be optically identified; it was identified with the brightest member of a cluster of galaxies (Mills and Thomas 1951; Baade and Minkowski 1954). Though Baade and Minkowski (1954) considered it to be two galaxies in collision, more recent research on the classification of galaxies has shown that Cyg A is a not untypical cD galaxy with a double nucleus (Matthews, Morgan, and Schmidt 1964; Bautz and Morgan 1970). It is possible that the double nucleus is an apparent result of the presence of a dust lane similar to that observed in the much nearer galaxy Cen A = NGC 5128, classified as DE3 by Matthews, Morgan, and Schmidt (1964).

The first spectrograms of Cyg A obtained by Baade and Minkowski (1954) showed that this object has a peculiar spectrum (for a galaxy), strong in forbidden emission lines, and with only a very weak continuum. Further observations by Schmidt (1965) showed that many, but by no means all, radio galaxies have these emission lines in their spectra, and that Cyg A may be regarded as a prototype of such objects. Its spectrum is qualitatively similar in many respects to the spectra of Seyfert galaxies (Baade and Minkowski 1954; Bautz and Morgan 1970), and the same types of lines are observed in many quasars (see, e.g., Burbidge and Burbidge 1967). Thus it is clearly important to understand the emission-line spectrum of Cyg A, not only

for itself, but because it is probably related to many of these other objects.

The available optical information on Cyg A, derived from photographic spectrograms taken at the Hale Observatories, has been collected and discussed by Mitton and Mitton (1972). Though their data are most interesting, they are necessarily qualitative, as they result from eye estimates made on uncalibrated spectrograms. In the present paper we report the results of our photoelectric spectrophotometry of Cyg A and the physical conclusions we can draw from it.

# II. OBSERVATIONAL PROCEDURES

All the observational data reported in this paper were obtained with the image-tube scanner (Robinson and Wampler 1972, 1973) at the Cassegrain focus of the 120-inch (3 m) telescope, in the years 1972–1974. All except the last two series of observations were taken in 1972 and 1973 with the original borrowed spectrograph mentioned in those papers, using a single (pierced) 600 lines mm<sup>-1</sup> grating. The two entrance apertures were set at  $0.70 \times 1$  mm, projecting to  $2.77 \times 4$  on the sky, separated by 20.6. In order to avoid faint stars in the comparison (sky) channel, the instrument was rotated so that the long axis of the slits was in position angle 11° for most of the measurements of Cyg A, and the telescope was guided visually using the television slit-viewing system. It can be seen from the direct photographs published by Baade and Minkowski (1954) that most, but by no means all,

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of the area of the bright double nucleus was covered by the slit.

With this instrument, a spectral region of somewhat over 2000 Å could be observed at one time, with a spectral resolution of about 6 Å. Therefore two alternate grating settings were used, one registering the spectrum from below  $\lambda 3727$  to above  $\lambda 5007$  ("blue"), the other from below  $\lambda 4861$  to above  $\lambda 6731$  ("red") in the rest wavelength system of Cyg A. A few of the blue measurements were taken with an image tube with an ultraviolet-transmitting fiber-optics plate; for these measurements, the grating setting was chosen so that the spectral region extended below  $\lambda 3346$  in the Cyg A rest wavelength system. An image tube with an ordinary fiber-optics faceplate and extended red sensitivity was used for the other blue measurements and for all of the red measurements.

The new Cassegrain spectrograph, designed specifically to be used with the image-tube image-dissector system (Miller, Robinson, and Wampler 1975), was used for one series of measurements extending below  $\lambda 3346$  ("ultraviolet") in 1974 July, as well as for one red series in 1974 September. In this instrument, each of the entrance apertures was also set at a size corresponding to 2".7 × 4".0, and their separation was 35". The throughput of light is considerably greater than in the older spectrograph, and the spectral region measured includes about 2200 Å, but in other details these measurements were similar to those with the older spectrograph.

On a typical night, Cyg A would be observed for a time between 32 minutes and 80 minutes, most often for 64 minutes. Measurements were made with the galaxy alternately in the two slits for times of 4 or 8 minutes; the blank sky was always measured at the same time in the other slit. Measurements of the blue spectral region were made on seven nights for a total of 336 minutes; of the red region, on three nights for a total of 192 minutes, and of the ultraviolet region with the new spectrograph, on one night for 64 minutes. The sensitivity of the system as a function of wavelength was determined for each night from measurements of standard hot stars chosen to have as few lines in their spectra as possible. One or more of the standard stars BD+28°4211, BD+33°2642, Hiltner 102, Feige 15, and Feige 25 was measured on each night, typically for a time of 4 minutes; and the measurements of these stars in energy units by Stone (1974), which in turn are based on the calibration of Hayes (1970), were thus transferred to Cyg A.

In addition, scans were taken each night of the line spectra of neon and helium comparison lamps, to determine the wavelength calibration (the mercury lines in the sky spectrum were also used for this purpose), and of the continuous spectrum of an incandescent lamp with a quartz bulb, to determine local variations in the sensitivity of the image-tube image-dissector system.

The measurements of Cyg A were then reduced with standard Lick Observatory PDP-8 programs, as outlined by Robinson and Wampler (1972, 1973) and by Miller, Robinson, and Wampler (1975), including sky

subtraction, reduction to a linear wavelength scale, correction for atmospheric extinction (using mean extinction coefficients measured at Mount Hamilton), and reduction to absolute energy units. The results are, for each emission line, the measured flux at the top of the Earth's atmosphere in ergs cm<sup>-2</sup> s<sup>-1</sup> Hz<sup>-1</sup>. Note that by our observation and reduction procedure, these fluxes are measured in the rest system of the Earth. Because of the finite redshift of Cyg A, a relativistic correction is necessary to find the fluxes that would be measured in the rest system of the galaxy; namely, if  $F_L(\lambda)$  is the flux measured in our rest system of a line with wavelength  $\lambda$ , then the flux of the same line in the rest system of Cyg A is

$$F_{L0}(\lambda_0) = (1 + z)^2 F_L(\lambda),$$

where

$$\lambda = (1 + z)\lambda_0.$$

Likewise the flux measured in the continuum in our rest system,  $F_{\nu}(\lambda)$ , is related to the flux in the continuum in the rest system of Cyg A,  $F_{\nu 0}(\lambda_0)$ , by

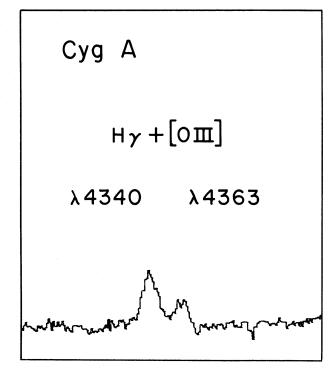
$$F_{\nu 0}(\lambda_0) = (1+z)F_{\nu}(\lambda).$$

Therefore, in all ratios of line strengths, or of continuum strengths, the factor for converting between the two systems cancels out, and it is only in calculating the flux in a particular line or at a particular point in the continuum that z must be used.

Note further that as a consequence of the finite resolution of the spectrograph (set largely by the slit width) and the finite widths of the emission lines in Cyg A, some of these lines were partly blended. These partial blends, examples of which are shown in figure 1, are H $\gamma$  and [O III]  $\lambda$ 4363; [O I]  $\lambda$ 6364 and [Fe x]  $\lambda$ 6374 (Minkowski and Wilson 1956); H $\alpha$  and [N II]  $\lambda\lambda$ 6548, 6583; and [S II]  $\lambda\lambda$ 6717, 6731. The measurements of these blends were decomposed into the individual line strengths as accurately as possible, using programs in which line profiles of unblended single lines are used as models, and scaled in intensity and width to best reproduce the overall profile. These comparisons can be rapidly made visually at the cathode-ray tube that is an integral part of the PDP-8 reduction system. On the other hand, the two components of  $[O II] \lambda \lambda 3726$ , 3729 and [N I]  $\lambda\lambda$ 5198, 5200 are so close together that these blends were treated as single lines throughout the reduction.

### III. MEASURED RESULTS

The widths of several of the stronger single emission lines in Cyg A were measured by numerically convolving the observed profile of a comparison line with Gaussian profiles of various assumed widths. The Cyg A lines measured included [O II]  $\lambda 3727$ , H $\beta$ , and [O III]  $\lambda 5007$ . For all these lines, the full width at half-maximum of the best-fitting Gaussian was  $500 \, \mathrm{km \, s^{-1}} \pm 100 \, \mathrm{km \, s^{-1}}$ ; no differences were found between the different lines beyond those attributable to differences in focus. (With the slit used for the measurements, the



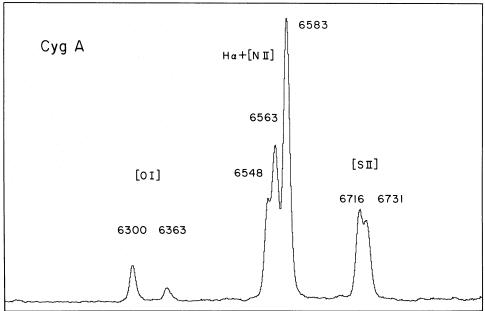


Fig. 1.—Sample single-night scans of Cyg A, reduced to energy units. Top, H $\gamma$  and [O III]  $\lambda$ 4363; bottom, region from [O I]  $\lambda$ 6300 through [S II]  $\lambda\lambda$ 6717, 6731. Note the asymmetry of  $\lambda$ 6363, probably due to [Fe x]  $\lambda$ 6374.

minimum detectable velocity width is about 300 km

s<sup>-1</sup>.)

The fluxes of all emission lines detectable on our spectral scan were measured as described in the previous section, and are listed, relative to the measured flux of H $\beta$ , in table 1. Each night's measurements were reduced independently, and in addition the reduced

scans of all the nights for each spectral region were averaged together. The first procedure gives an estimate of the accidental errors of the measurements, while the second procedure improves the signal-tonoise ratio and gives the best opportunity for detecting faint lines. The accidental probable errors of the ratios listed in table 1 range from about 5 percent to

TABLE 1
OBSERVED RELATIVE LINE INTENSITIES IN CYGNUS A

	Measured		Corrected	
Line	$F/F(H\beta)$ 1	$\log F/F(H\beta)$	$I/I(H\beta)$	$\log I/I(H\beta)$
[Ne v] 3346   [Ne v] 3426   [O II] 3727   [Ne III] 3868   [Ne III] 3969   [S II] 4071   Hδ 4101   Hγ 4340   [O III] 4363   He I 4471   He II 4686   Hβ 4861   [O III] 4959	0.14 0.36 2.44 0.66 0.22 0.14 0.17 0.32 0.16 ≤0.07 0.25 1.00 4.08	$ \begin{array}{r} -0.86 \\ -0.44 \\ +0.39 \\ -0.18 \\ -0.65 \\ -0.86 \\ -0.76 \\ -0.48 \\ -0.81 \\ \leq -1.15 \\ -0.61 \\ 0.00 \\ +0.61 \end{array} $	0.38 0.95 5.00 1.23 0.40 0.23 0.28 0.46 0.21 ≤0.09 0.28 1.00 3.88	$ \begin{array}{r} -0.42 \\ -0.02 \\ +0.70 \\ +0.09 \\ -0.40 \\ -0.64 \\ -0.55 \\ -0.34 \\ -0.68 \\ \leq -1.05 \\ -0.56 \\ 0.00 \\ +0.59 \end{array} $
[O III]	$\begin{array}{c} 13.11 \\ 0.40 \\ \leq 0.10 \\ \leq 0.10 \\ \leq 0.10 \\ 0.14 \\ 0.13 \\ \leq 0.07 \\ 2.10 \\ 0.69 \\ 0.10? \\ 3.94 \\ 6.61 \\ 13.07 \\ 3.65 \\ 3.29 \\ \end{array}$	$\begin{array}{l} +1.12\\ -0.40\\ \leq -1.00\\ \leq -1.00\\ -0.86\\ -0.89\\ \leq -1.16\\ +0.32\\ -0.16\\ -0.99\\ +0.60\\ +0.82\\ +1.12\\ +0.56\\ +0.52\\ \end{array}$	$\begin{array}{l} 12.30 \\ 0.32 \\ \leq 0.08 \\ \leq 0.06 \\ 0.09 \\ 0.08 \\ \leq 0.05 \\ 1.10 \\ 0.35 \\ 0.05 \\ 1.90 \\ 3.08 \\ 6.15 \\ 1.66 \\ 1.51 \\ \end{array}$	$\begin{array}{l} +1.09\\ -0.49\\ \leq -1.11\\ \leq -1.19\\ -1.05\\ -1.11\\ \leq -1.42\\ +0.04\\ -0.45\\ -1.28\\ ?\\ +0.28\\ +0.49\\ +0.79\\ +0.22\\ +0.18\\ \end{array}$

about 20 percent, depending on the strength of the line and its wavelength separation from H $\beta$  and [O III]  $\lambda\lambda$ 4959, 5007. The systematic errors resulting from the remaining uncertainties in the sensitivity of the instrument as a function of wavelength are much less easy to determine, but they probably are of the general order of 10 percent.

In addition to the lines that were detected in the Cyg A scans, a determined effort was made to find, or set upper limits to, the fluxes of other lines that might conceivably be present in the spectrum. Mitton and Mitton (1972) listed a large number of lines that they thought were seen on one or more of the photographic spectrograms of Cyg A, and we searched our summed scans for several of the more plausible of these lines, as well as for other lines that, on physical grounds, might possibly be expected. Upper limits are given in table 1 for several of the astrophysically most important lines that were not detected. For single unblended lines, this upper limit was found by locating the redshift position where the line would be, if present on the spectrum, and either measuring whatever line is apparently present at that position, or estimating how strong a line might be present that would still be interpreted as noise rather than as a line. The [Fe x] λ6374 reported as present by Minkowski and Wilson (1956) is probably, but not certainly, present; the evidence is a slight asymmetry of  $\lambda 6364$  toward longer wavelength in both of the longexposure red scans, plus the fact that this asymmetric line is stronger with respect to [O I]  $\lambda 6300$  than

expected for [O I]  $\lambda 6364$  from the relative transition probabilities.

In addition to the relative fluxes listed in table 1, we of course also measured the absolute flux in each line. However, since the slit widths used with both Cassegrain spectrographs are narrower than the bright double nucleus of Cyg A, the measurements do not include all the light of the object. Our absolute calibration is based on a measurement made in 1968 with the original prime-focus scanner (Wampler 1968a) on the 120-inch telescope, using a circular aperture with diameter 9".4. The flux in [O III]  $\lambda$ 5007, the strongest line measured in Cyg A, within this diameter is  $F_L(\lambda 5007) = 1.58 \times 10^{-13}$  ergs cm<sup>-2</sup> s<sup>-1</sup>, with a probable error of about 10 percent. Using the ratio  $F(\lambda 5007)/F(H\beta)$  given in table 1, this corresponds to  $F_L(H\beta) = 1.20 \times 10^{-14} \text{ ergs cm}^{-2} \text{ s}^{-1}$ . We have adopted a redshift z = 0.0566, the mean of the values given by Schmidt (1965) and Sandage (1972), leading to the flux in H $\beta$  in the rest system of Cyg A (at the Earth's position in space)  $F_{L0}(H\beta) = 1.34 \times 10^{-14} \text{ ergs cm}^{-2} \text{ s}^{-1}$ .

Though the interpretation of these measurements will be discussed in the next section, we may note here that the Balmer decrement exhibited in table 1 is steeper than can occur in recombination, or indeed under any plausible physical conditions in an extended gas cloud, and can reasonably be interpreted as resulting from interstellar extinction (see Wampler 1968b; Osterbrock 1971). Therefore, to correct for this effect, we have in the absence of other information assumed the standard Whitford (1958) interstellar extinction curve in the rest system of Cyg A. The amount of extinction adopted was chosen to give the best fit between the observed Balmer decrement and the computed Balmer decrement (Brocklehurst 1971) for  $T=10^4$  K,  $N_e=10^4$  cm<sup>-3</sup>, though the result depends only slightly on the assumed temperature and practically not at all on the density. Using the tabulation of the Whitford extinction curve given by Osterbrock (1974), the constant c giving the amount of extinction (in logarithms to the base 10) was determined as c = 1.03 from H $\alpha$ /H $\beta$ , c = 1.00 from H $\alpha$ /H $\gamma$ , and c = 0.82 from H $\alpha$ /H $\delta$  (the last poorly determined because of the weakness of  $H\delta$ ). We adopt c = 0.95, corresponding to a differential extinction of  $\Delta[\log F_{L0}(\mathrm{H}\gamma)/F_{L0}(\mathrm{H}\alpha)] = 0.475$ , or  $\Delta\log(F_B/F_V) = 0.28$  or  $E_{B-V} = 0.69$  mag. This is considerably larger than the extinction  $E_{B-V} = 0.30$  derived for Cyg A by Sandage (1972), but his value is open to question, as it is based on color measurements and the assumption that Cyg A has the spectrum of a normal E galaxy, while the observations show that the emission lines are very strong and that absorption lines are almost undetectable in its continuous spectrum (see below). We have therefore corrected all the line ratios of table 1 for interstellar extinction, using the extinction derived from the Balmer emission-line decrement, and these corrected ratios are also listed in the same table.

The fluxes in the continuum were also measured from the spectral scans of Cyg A. The continuous

TABLE 2

	OBSERVED CONTINUUM INTEN	SITIES IN CYG	Α
Me	easured	Corr	ected
λ	F <sub>ν</sub> /F <sub>ν</sub> (λ5135)	λ <sub>o</sub>	F <sup>0</sup> <sub>ν</sub> /F <sup>0</sup> <sub>ν</sub> (λ4861)
3470	0.199 0.226	3284 3388	0.565 0.602
3580 3700	0.223	3502	0.566
3800	0.256	3596	0.585
4000	0.310	3786	0.623
4140	0.286	3918	0.505
4235	0.401	4008	0.686
4410	0.396	4174	0.600
4520	0.489	4278	0.707
4690	0.595	4439	0.771
4855	0.785	4595	0.954
5010	0.965	4742	1.029
5070	0.918	4798	0.958
5171	1.085	4894	1.083
5185	1.16	4907	1.131
5388	1.14	5099	0.996
5410	1.25	5120	1.078
5685	1.46	5380 5555	1.121 1.258
5870	1.79 1.95	5813	1.288
6142	2.26	5996	1,368
6335 6519	2.15	6170	1.162
6772	2.38	6409	1.181
7025	3.12	6649	1.418
7237	2.96	6849	1.260

spectrum is quite faint, and therefore it was measured only on the summed scans of all available data for each spectral region. The measurements were made on the analog tracings, averaging by eye over regions approximately 30 Å wide, chosen to be in clear regions free of any emission lines. No absorption lines at all are visible on our scans, and in particular, as we do not see either a Ca II  $\lambda$ 3933 K-line or a Mg I  $\lambda$ 5175 b-blend, we estimate upper limits to the content of normal elliptical-galaxy integrated stellar spectrum at these two wavelengths to be about 25 percent. The final values of the observed fluxes  $F_{\nu 0}$  in the continuum in Cyg A, found by averaging all the scans covering each

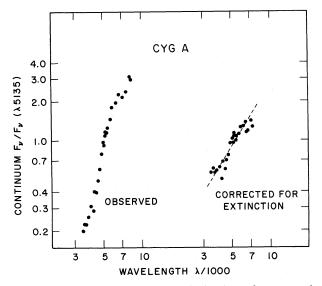


Fig. 2.—Continuous spectrum of Cyg A. Left, measured continuum; right, continuum corrected for interstellar extinction as described in text. The dashed line represents a power-law spectrum with exponent n = 1.6.

spectral region, are listed in table 2, and plotted in figure 2, expressed as ratios to  $F_{\nu 0}(\lambda 5135)$ , the continuum flux at  $\lambda 5135$  (the redshifted position of H $\beta$ ). From the internal consistency of the independent measures, and from the noise in the tracings, we estimate that the probable error of each continuum point is about  $\pm 20$  percent. Note that the lowest point with respect to the line is at the redshifted wavelength corresponding to  $\lambda_0 = 3920$  Å, but no sign of a Ca II K absorption line can be seen; the tracing appears smooth in this region.

In the case of the continuum, there is of course no independent information on its intrinsic color, and we have therefore corrected it for interstellar extinction assuming the same amount of extinction derived and used above for the emission-like spectrum. This is necessarily rather speculative and could be greatly in error, but there is simply no other assumption that seems more reasonable than this one. The continuum fluxes corrected for extinction in this way are also listed in table 2 and plotted in figure 2. It can be seen in this figure that the corrected continuum can be approximately fitted with a power law within the probable error of the observations. The spectral index for the continuum,

$$\frac{F_{\nu}(\lambda)}{F_{\nu}(\lambda5135)} = \frac{F_{\nu0}(\lambda_0)}{F_{\nu0}(\lambda4861)} = \frac{(\nu_0)^{-n}}{(\nu_{\lambda4861})},$$

is approximately  $n=1.6\pm0.2$ . This is considerably less steep than the index n=3.2 reported by Oke (1968), no doubt largely because he applied a considerably smaller correction for extinction based on the colors of normal elliptical galaxies in the field around Cvg A.

The absolute value of the flux in the continuum was determined by measuring the equivalent width of the emission line H $\beta$  on several of the averaged scans of Cyg A. The result is (in the rest system of the Earth)  $W_{\lambda}(H\beta) = 41.2 \text{ Å} \pm 2 \text{ Å}$ ; so, using the measured flux in H $\beta$  quoted above, the flux in the continuum is  $F_{\nu}(\lambda 5135) = 2.56 \times 10^{-27} \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}$  at the redshifted position of H $\beta$ . The corresponding flux in the rest system of Cyg A is  $F_{\nu 0}(\lambda 4861) = 2.70 \times 10^{-27} \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}$ . The equivalent width of H $\beta$  in the rest system of Cyg A is  $W_0(H\beta) = W_{\lambda}(H\beta)/(1+z) = 39 \text{ Å}$ .

Finally, we may correct the absolute values of the line and continuum fluxes for interstellar extinction, but here it must be realized that the correction depends upon the extrapolation of the extinction curve to infinite wavelength (zero extinction), and upon the assumption that the relation between extinction and reddening is the same in Cyg A as in the region of the Galaxy in which the Whitford curve was determined. These assumptions may not be true, but they seem the best we can make with our present knowledge. The amount of extinction determined from the Balmer decrement, under these assumptions, corresponds to an extinction of 1.04 in the logarithm, or 2.6 mag, at H $\beta$ . Thus the flux in H $\beta$ , corrected for interstellar extinction and measured in the rest system of Cyg A, is

 $F^0{}_{L0}({\rm H}\beta)=1.46\times 10^{-13}~{\rm ergs~cm^{-2}~s^{-1}},$  while the continuum flux at the same wavelength is  $F^0{}_{\nu0}(\lambda 4861)=2.94\times 10^{-26}~{\rm ergs~cm^{-2}~s^{-1}~Hz^{-1}}.$  These values, together with tables 1 and 2, give the best available observational values of the fluxes emitted by Cyg A in the emission lines and continuum.

Note that the extinction adopted here corresponds to extinction in the UBV system of  $A_B = 2.85$  mag and  $A_V = 2.26$  mag, rather than the values  $A_B = 1.20$  and  $A_V = 0.90$  adopted by Sandage (1972). It therefore increases the derived absolute magnitude of Cyg A, corrected for reddening, to  $M_{VC} = 24.9$ , nearly 1.5 mag brighter than the value Sandage (1972) adopted. However, convolution of our measured energy distribution with published B and V response curves indicates that the emission lines make B numerically more negative by 0.3 mag, and V by 0.5 mag, than the values they would have if the Cyg A continuum alone were measured. This same convolution gave a computed B - V = 1.42 for Cyg A, which is very close to the measured value B - V = 1.45 (Sandage 1972), and thus furnished a good check of the spectrophotometry.

### IV. INTERPRETATION

A qualitative description of the spectrum of Cyg A listed in table 1 is that it contains the typical recombination lines of a fairly high-ionization planetary nebula, plus strong forbidden lines extending over a wide range of ionization, with unusually strong lines of once-ionized ions and neutral atoms. No planetary nebulae or H II regions are known with [N II]  $\lambda 6583$  stronger than H $\alpha$ ; [S II]  $\lambda \lambda 6717$ , 6731 and [O I]  $\lambda 6300$  stronger than H $\beta$ ; and [N I]  $\lambda 5199$  as strong as H $\gamma$ , as in Cyg A. Likewise, only a few planetaries have spectra with the high-ionization lines [Ne V]  $\lambda \lambda 3346$ , 3426 which are certainly present in Cyg A, and none are known to have [Fe x]  $\lambda 6374$ , probably present in Cyg A.

At the more quantitative level, the observed H I lines can be corrected for interstellar extinction so that they fit the calculated recombination Balmer decrement for Case B conditions (Brocklehurst 1971) reasonably well, as described in the previous section. The observed strengths of He I  $\lambda$ 5876 and He II  $\lambda$ 4686 may therefore be used to find the relative abundances of He<sup>+</sup> and He<sup>++</sup> from the standard recombination-line formulae

$$\frac{I(\lambda 5876)}{I(H\beta)} = \frac{N(He^+)}{N(H^+)} \frac{\alpha_{\lambda 5876}}{\alpha_{H\beta}} \frac{h\nu_{\lambda 5876}}{h\nu_{H\beta}}$$

and

$$\frac{I(\lambda 4686)}{I(H\beta)} = \frac{N(He^{++})}{N(H^{+})} \frac{\alpha_{\lambda 4686}}{\alpha_{H\beta}} \frac{h\nu_{\lambda 4686}}{h\nu_{H\beta}} \cdot$$

Using the numerical values of the effective recombination coefficients calculated by Brocklehurst (1971, 1972) for  $T=10^4$  K,  $N_e=10^4$  cm<sup>-3</sup> (though the results are quite insensitive to temperature and density), the results are  $N({\rm He^+})/N({\rm H^+})=0.057$ ,  $N({\rm He^+}^+)/N({\rm H^+})=0.024$ , and hence  $[N({\rm He^+})+N({\rm He^+}^+)]/N({\rm H^+})=0.08$ . The observational uncer-

tainty of  $N(\text{He}^+)/N(\text{H}^+)$ , which is based on the weak line  $\lambda 5876$ , is probably about  $\pm 20$  percent. The upper limit to He I  $\lambda 4471$  is consistent with any He<sup>+</sup> abundance  $N(\text{He}^+)/N(\text{H}^+) \leq 0.2$ , and hence gives no additional information. The observed abundance ratio  $[N(\text{He}^+) + N(\text{He}^{++})]/N(\text{H}^+) = 0.08 \pm 0.02$  is thus a little smaller than the ratio  $N(\text{He})/N(\text{H}) = 0.11 \pm 0.01$  found in very many planetary nebulae and H II regions (see Osterbrock 1974 for a recent compilation from many sources). Note, however, that the great strength of  $[O\ I]$ ,  $[N\ I]$ ,  $[S\ II]$ , and  $[N\ II]$  in Cyg A shows that there are large regions of relatively low ionization in which He<sup>0</sup> may be less ionized than H<sup>0</sup>, and hence a more nearly accurate observational statement is  $N(\text{He})/N(\text{H}) \geq 0.08 \pm 0.02$ .

Some diagnostic mean values can be derived directly from observed forbidden-line ratios. The [O III] ratio  $[I(\lambda4959) + I(\lambda5007)]/I(\lambda4363) = 77$  implies a mean temperature  $T = 15,000 \pm 1000$  K in the region of [O III] emission, while the [N II] ratio  $[I(\lambda6548) + I(6583)]/I(\lambda5755) = 90$  implies a mean  $T = 10,000 \pm 1000$  K in the [N II] emitting region. (The formulae and necessary numerical values for these well-known methods are taken from the compilation in Osterbrock 1974.) These two observational determinations are of course not necessarily inconsistent, because the regions of N+ and O++ may have relatively little overlap.

The [S II] ratio  $\lambda 6717/\lambda 6731 = 1.10$  corresponds to a mean electron density  $N_e(10^4/T)^{1/2} = 8 \times 10^2$  cm<sup>-3</sup> in the region of strong [S II] emission. The intensity ratio [S II]  ${}^{4}S-{}^{2}P$   $\lambda 4071$  (actually a blend of  $\lambda\lambda 4069$ , 4076) to  $[S II]^4S^{-2}D$   $\lambda\lambda6716$ , 6731 depends on both T and  $N_e$ , and assuming T=10,000 K determined from [N II] and  $N_e=8\times10^2$  cm<sup>-3</sup> from [S II], the calculated value of this ratio is  $I({}^{4}S-{}^{2}P)/I({}^{4}S-{}^{2}D) =$ 0.097, using the collision strengths and tables of Saraph and Seaton (1970), while the observed ratio from table 1 is 0.076. Considering the weakness of λ4071, this is quite good agreement; while if we adopt the measured ratio as exact, the calculated mean temperature in the [S II] emitting region is 8600 K. Probably a somewhat lower temperature should be expected in the S<sup>+</sup> zone than in the N<sup>+</sup> zone, because the ionization potential of So is only 10.4 eV; so if photoionization is responsible for the ionization, [S II] emission occurs in a partly ionized region where the energy input and hence the temperature is lower than in the main [N II] region, while if the emission comes from a recombining region, the temperature of the peak abundance of S<sup>+</sup> is lower than for N<sup>+</sup>.

The mass of ionized gas in Cyg A may be estimated from the observed Balmer-line flux. The observed flux in H $\beta$ , corrected for interstellar extinction, gives the luminosity in H $\beta$ ,

$$L(H\beta) = 4\pi D^2 F_{L0}^0(H\beta) = 2.0 \times 10^{42} \text{ ergs s}^{-1}$$
,

where the distance  $D = 3.4 \times 10^8$  pc is derived from the redshift z = 0.0566 quoted above and the Hubble constant  $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (Sandage 1973). Then since

$$L(H\beta) = N_p N_e \alpha_{H\beta} V,$$

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where V is the volume of the ionized gas; if we adopt  $T=10^4$  K,  $N_e=10^3$  cm<sup>-3</sup> as representative mean values for the ionized gas, we find

$$N_pV = 1.6 \times 10^{64}$$
 protons

or

$$M = \mu N_p V = 2 \times 10^7 M_{\odot}$$

as the approximate mass of ionized gas in Cyg A, adopting a mean mass per proton  $\mu = 1.4 m_{\rm H}$ , corresponding to  $N({\rm He})/N({\rm H}) = 0.1$ . The numerical value of the mass of ionized gas is inversely proportional to the square root of the adopted value of T.

Furthermore, with the same assumed mean value  $N_p \approx N_e \approx 10^3 \, \mathrm{cm}^{-3}$ , the ionized volume corresponds to a sphere with radius 50 pc. At the distance of Cyg A, such a sphere would subtend a diameter of approximately 0.06, far smaller than the observed light central region which Baade and Minkowski (1954) described as approximately  $3'' \times 5''$ . Evidently in Cyg A, as in many other emission-line galaxies, the ionized gas has a very patchy or filamentary distribution within the observed volume or, put another way, the filling factor is quite small.

The wide range of ionization observed in the emission-line spectrum of Cyg A shows that a fairly detailed model, based on a consistent physical picture, will be necessary to determine from the observations the physical conditions in this object. Agreement of the calculated spectrum of a model with the observed spectrum will be a necessary but not sufficient condition for its correctness. Such a program is beyond the scope of the present paper, but we can, from the observations themselves, draw some rough estimates of the physical conditions as starting points for more detailed models.

Let us first discuss the abundances. From the observed relative line strengths, relative abundances of the ions responsible for the emission of observed lines can be determined from the standard well-known methods and formulae of nebular astrophysics summarized, for example, by Osterbrock (1970). In the present discussion we have adopted mean temperatures T = 12,000 K for the Balmer lines presumably emitted throughout the ionized volume; T = 8500 K for the [S II], [O I], and [N I] lines emitted in the regions of lowest ionization; T = 10,000 K for the [N II] and [O II] lines; T = 15,000 K for the [O III] and [Ne III] lines; and T = 20,000 K for the [Ne v] and [Fe vII] lines. To calculate the emission coefficients, the Case B recombination-theory values of Brocklehurst (1971, 1972) were used for the H I, He I, and He II lines, and collision strengths calculated by Seaton and his collaborators were used for the forbidden lines. (See Osterbrock 1974 for numerical values and references for the individual ions.) A density  $N_e = 800 \text{ cm}^{-3}$ was assumed when necessary to take collisional deexcitation into account. The resulting relative abundances are collected in table 3; the probable errors are necessarily large and uncertain because the physical picture is so schematic. Note that not all these ions are assumed to coexist at one point in Cyg A; the abun-

TABLE 3
RELATIVE IONIC ABUNDANCES IN CYGNUS A
EMISSION-LINE REGION

Ion	Abundance	Ion	Abundance
H+ He+ He++ N <sup>0</sup> N+	$5.7 \times 10^{2}$ $2.4 \times 10^{2}$ 0.37	O <sup>0</sup> O <sup>+</sup> O <sup>++</sup> Ne <sup>++</sup> Ne <sup>+4</sup> Fe <sup>+6</sup>	1.7 1.5 0.45 0.16

dances quoted rather are a sum or average over the entire volume observed, defined by the slit, the depth of the ionized gas in the line of sight, and the extinction due to dust. Making some approximate allowance for unobserved stages of ionization (including H<sup>0</sup>, which is almost certainly present along with the observed O<sup>0</sup>), the relative atomic abundances can be roughly estimated as listed in table 4. These values are, as stated above, not very well determined, but may be useful starting points for model computations. These abundances are not very different from values given in modern abundance compilations, particularly those derived from nebular observational data, as for example those summarized by Miller (1974a). The conclusion is that in Cyg A there are no clearly abnormal abundances among the elements listed in table 4.

Next let us discuss the possible mechanisms of energy input to the ionized gas. One possibility is the conversion of kinetic energy to heat—that is, shockwave heating such as occurs in the late stages of supernova remnants—while another possibility is photoionization by ultraviolet radiation. The great strength of [O I] and [S II] lines in Cyg A (and some other radio galaxies), similar to their strengths in the Cygnus Loop (Miller 1974b) and N49, suggests that shock heating may be an important mechanism (Osterbrock and Dufour 1973). However, the weakness of [O III]  $\lambda 4363$  in Cyg A, and the resulting low calculated mean temperature in the [O III] emitting region, T =15,000 K, shows that the input is not completely due to direct input of heat energy derived from kinetic energy (Cox 1972). It is possible that because the mass motion velocities are higher in Cyg A than in the supernova remnants (of order 500 km s<sup>-1</sup> versus 150 km s<sup>-1</sup>), enough ultraviolet radiation is emitted in the shock-heated region (Nussbaumer and Osterbrock 1970) that the [O III] region produced by photoionization is larger and more luminous than the [O III] region produced by recombination behind the shock, which would explain this observation. Models

TABLE 4

Approximate Elemental Abundances in Cygnus A

Emission-Line Region

Element Abundance	Element Abundance
H10 <sup>4</sup> He10 <sup>3</sup> N1 O4	Ne1 S0.3 Fe≤0.1

of high-velocity shock waves are necessary to test this possibility quantitatively, and work in this direction by Cox, Daltabuit, and MacAlpine is in progress (see MacAlpine 1971).

The other possibility is that the input of energy occurs by photoionization by ultraviolet radiation. As noted above, the great strengths of the lines of [O I], [N I], and [S II] show conclusively that the ultraviolet radiation is not derived from any known kind of star, as in planetary nebulae and HII regions. However, the possibility remains open of a synchrotron continuum, emitted by a source within Cyg A; for, as Mitton and Mitton (1972) have stated, the observed wide range of ionization in Cyg A is similar to that predicted by photoionization models with relatively high-energy input spectra. They found the best match with a model calculated by Bergeron and Souffrin (1971) that is partly ionized by 200 eV photons emitted throughout the volume, rather than by a "point" source. However, only relatively few line strengths were calculated for this model; in addition, some physical effects such as charge-exchange reactions were not taken into account, so this model cannot really be thoroughly tested.

We have compared the observed Cyg A line intensities with a photoionization model originally calculated by MacAlpine (1971) as a possible interpretation of the emission-line spectra of Seyfert galaxies. This model is calculated for an input spectrum (from an assumed central source) with a power-law dependence  $F_{\nu} \propto \nu^{-n}$  with n=1.2, not too different from the index determined for the optical continuum of Cyg A,  $n=1.6\pm0.2$ . MacAlpine's model further assumes an electron density approximately  $N_e=10^4~{\rm cm}^{-3}$ , a filling factor  $\epsilon=0.01$ , a helium abundance  $N({\rm He})/N({\rm H})=0.06$ ; the model is terminated at a radius of approximately 40 pc, twice the radius of the calculated H<sup>+</sup> zone. (It is Model 1-B of the thesis, which should be consulted for further details.)

The comparison is given in table 5, where the first column repeats the observed Cyg A relative line intensities  $I/I(H\beta)$ , corrected for interstellar extinction from table 1, and the next column lists the predicted values according to this model. It can be seen that the observations and predictions agree in a general way, though the agreement is far from perfect. Note, however, that the observed He I and He II lines are all stronger than the model predicts; the agreement could be improved by adopting a somewhat higher He abundance in the model. Likewise the [Ne III] and [Ne v] lines are all observed stronger than the model predicts; the agreement here could also be considerably improved by adopting a larger Ne abundance for the model. There is a large discrepancy between the observed and predicted [O II]  $\lambda 3727$  intensities, but a considerable part of this is due to the high electron density assumed in the model and consequent collisional descritation of O<sup>+</sup>. Reducing the assumed density from  $N_e = 10^4$  cm<sup>-3</sup> to  $10^3$  cm<sup>-3</sup> suggested by the [S II] line ratio would increase the calculated [O II]  $\lambda 3727$  intensity from 0.24 to 0.6, which is lower than the observed ratio, but closer to it.

TABLE 5
RELATIVE LINE INTENSITIES

Ion	λ(Å)	Cygnus A	MacAlpine Model	Crab Nebula	NGC 4151 (omitting wings)
[Ne v]	.3346	0.38	0.12		
[Ne v]		0.95	0.34	0.27	1.8
[О п]		5.00	0.24	12.6	2.8
Ne III		1.23	0.53	1.90	1.6
[Ne III]		0.40	0.16		
[S II]		0.23			0.45
Ηδ		0.28	0.26	0.31	0.28
Ηγ		0.46	0.47	0.61	0.39
[О́ ш]		0.21	0.19	0.19	0.39
Не 1		≤0.09	0.02	0.28	0.05:
Не п	.4686	0.28	0.18	0.68	0.39
$H\beta$	.4861	1.00	1.00	1.00	1.00
[О ш]	.4959	3.88	6.3	3.92	3.9
[О ш]	. 5007	12.30	18.1	11.92	11.9
[N I]	.5199	0.32			
[Fe xiv]	. 5303	≤0.08	0.01		0.05:
[Fe vII]	.5720	≤0.06	0.03		0.22
[N II]	. 5755	0.09		0.11	0.17:
Не 1	. 5876	0.08	0.06	0.79	0.22
[Fe VII]	.6087	≤0.04	0.04	·	0.39
[0]	. 6300	1.10	1.24	1.20	1.00
[0 1]	. 6364	0.35	0.41	0.33	0.22
[Fe x]	.6374	0.05?	0.07		0.11:
[N II]		1.90	0.29	1.36	0.28
Ηα		3.08	2.85	3.16	1.61
[N II]	.6583	6.15	0.86	4.10	1.39
[S II]		1.66	·}	9.24	1.33
[S π]		1.51	∫	9.24	1.56

Note that even the probably observed [Fe x]  $\lambda 6374$  line is predicted by the photoionization model with n = 1.2 to have approximately the measured intensity ratio, though it presumably would be calculated to be much fainter for a model with the observed index, n = 1.6.

Since the photoionization model approximately reproduces the observed line intensities, it is of interest to discuss whether the observed optical continuum, extrapolated into the ultraviolet, can provide enough ionizing photons to explain all the observed ionization of Cyg A. The steady-state photoionization equation for hydrogen,

$$\int_{\nu_0}^{\infty} \frac{L_{\nu}}{h\nu} d\nu = \alpha_B N_e N_p V,$$

(where  $L_{\nu}$  is the luminosity of the source per unit frequency interval,  $\alpha_B$  is the effective recombination coefficient,  $h\nu_0$  is the ionization potential of H, and V is the ionized volume), together with the equation for the H $\beta$  luminosity of the ionized gas,

$$L_{\mathrm{H}\beta} = \alpha_{\mathrm{H}\beta} h_{\nu} \mathrm{H}\beta N_{p} N_{e} V,$$

can easily be solved for a power-law source with  $L_{\nu} \propto \nu^{-n}$ . The result, expressed in terms of  $W_0(H\beta)$ , the equivalent width of the H $\beta$  emission line in wavelength units, is

$$W_0(\mathrm{H}\beta) = \frac{\lambda_{\mathrm{H}\beta}}{n} \frac{\alpha_{\mathrm{H}\beta}}{\alpha_B} \left(\frac{\nu_0}{\nu_{\mathrm{H}\beta}}\right)^{-n} \cdot$$

Substituting numerical values (Brocklehurst 1971), we find  $W_0(H\beta) = 24$  Å, while the observed equivalent width (in the rest system of Cyg A) is 39 Å. Thus the observed continuous spectrum, extrapolated beyond the Lyman limit, can account for about two-thirds of the observed ionization, but not all. On the other hand, only a slight decrease of the index would force agreement; for instance, with n = 1.4,  $W_0(H\beta) = 39$  Å, the observed value, and any smaller assumed index would imply that some of the ionizing photons escape completely.

Furthermore, the energy radiated in collisionally excited lines plus free-free emission from the ionized gas is, in the photoionization model, all derived from the excess energy of the absorbed photons above the ionization potential of H. This condition leads to the relation

$$\sum \frac{I(\text{collisionally excited lines})}{I(H\beta)} \leq \frac{1}{(n-1)} \frac{\nu_0}{\nu_{H\beta}} \frac{\alpha_B}{\alpha_{H\beta}} \cdot$$

Again inserting numerical values, for n = 1.6 we find that the right-hand side is 76, approximately twice as large as the sum of the observed lines listed in table 1. Thus there is sufficient photoionization energy available in this model for the observed lines, though a calculation including lines in the unobservable ultraviolet would be necessary to confirm this point.

violet would be necessary to confirm this point. One further point of interest is the continuous spectrum of Cyg A over a large range of frequency, between the radio and optical regions. Hargrave and Ryle (1974) have observed Cyg A with high resolution at 5 GHz, and found that in addition to the two well-known bright components there is a weak central component, centered to very high precision in the middle of the optical nucleus of Cyg A, with measured flux density  $1.1 \text{ Jy} = 1.1 \times 10^{-23} \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}$ . Comparing this with the measured  $F_{\nu}^{0}(\lambda 5135) = 2.80 \times 10^{-26} \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} \text{ leads to a spectral index } n = 0.5 \text{ between the radio-frequency and optical regions. However, as Hargrave and Ryle (1974) note, the radio source has such a small angular size (<1") that it may be quite different physically from the optical source (<math>\sim 3'' \times 5''$ ), and hence this "spectrum" may have little physical meaning.

If the probably observed [Fe x]  $\lambda$ 6374 emission line is not attributed to photoionization by high-energy photons in the input power-law spectrum, it must indicate the presence of a very high temperature region in Cyg A, as Mitton and Mitton (1972) have suggested. If, to calculate the emission coefficient (Nussbaumer and Osterbrock 1970), we assume the most favorable temperature for collisional ionization to Fe<sup>+9</sup> and production of [Fe x]  $\lambda$ 6374,  $T \approx 2 \times 10^6$ , and approximate pressure equilibrium between this hot gas and the gas in which the other lines (of H I through Ne v) are emitted, the electron density in the very hot gas is calculated to be approximately  $N_e = 10^2$ , its volume approximately  $0.5 \times 10^{65}$  cm<sup>3</sup>, its mass approximately  $10^8 M_{\odot}$ , and its radius approximately 700 pc or 0".4—all in good agreement with the estimates of Mitton and Mitton (1972).

In table 5 the observed line intensities in Cyg A are compared not only with the photoionization model calculated by MacAlpine (1971), but also with observed line intensities for the Crab Nebula (NGC 1952) and the Seyfert galaxy NGC 4151. The column marked "Crab Nebula" is based on scanner measurements by Miller (1975), corrected for interstellar extinction from observed line ratios; the figures listed are straight averages between a measured "red" filament and a measured "green" filament, and thus more or less represent what would be observed in an integrated spectrum of the entire Crab Nebula, from a distance so large that none of the individual filaments could be resolved. There are good reasons for believing that the Crab Nebula is photoionized by a synchrotron continuum (Woltjer 1958; Williams 1967; Davidson 1973), so the comparison of the observed Cyg A spectrum with the observed Crab Nebula spectrum eliminates some of the uncertainties in the theory of photoionization models, mostly connected with density fluctuations, filamentary structure, and deviations from spherical symmetry. It can be seen that there is generally quite good agreement, except that the He I and He II lines are much weaker in Cyg A than in the Crab Nebula. This is of course a result of the high He abundance in the Crab Nebula (Woltjer 1958). Except for these differences, Cyg A appears to have an emission-line spectrum rather similar to that of the Crab Nebula, and hence may, like the Crab Nebula, be ionized by a synchrotron continuum extending to high energies in the extreme ultraviolet.

One further comparison is shown in table 5, between the emission-line spectra of Cyg A and the Seyfert galaxy NGC 4151, as measured and corrected for interstellar extinction by Oke and Sargent (1968). We have omitted the contributions of the broad wings of the Balmer lines to the intensities in NGC 4151, since these wings presumably are emitted in much denser regions than the forbidden lines and the cores of the Balmer lines, which have profiles similar to the forbidden lines (Oke and Sargent 1968). The He II λ4686 line also has broad wings in NGC 4151, but Oke and Sargent (1968) do not give the intensities separately for the core and wings, so the total intensity listed in table 5 for this line is actually an upper limit. It can be seen that the agreement between Cyg A and NGC 4151 is reasonably good, and in particular the [Ne III] and [Ne v] line intensities match better, while the [N II] intensities in NGC 4151 are considerably fainter than in Cyg A. It seems probable that the same process is responsible for the ionization and input of energy to the ionized gas in Cyg A as to the ionized gas that emits the "cores" or "sharp lines" in Seyfert galaxies like NGC 4151.

In summary, the measured emission-line intensities in Cyg A show a very wide range of ionization, from neutral atoms to [Ne v] and probably [Fe x]. The temperature measured from the [O III] lines is in the same general range as in planetary nebulae and H II regions, and this suggests that a large fraction of the input of energy to the ionized gas is by photoionization by a spectrum extending to very high energies. The

continuous spectrum of Cyg A shows no stellar absorption lines, and can be approximately matched to a power-law spectrum. Photoionization models based on input spectra not too different from extrapolations into the ultraviolet of the observed continuum can approximately reproduce the observed emission-line intensities. The observed spectrum is also similar to the Crab Nebula (except for He and probably also Ne abundance effects), which is quite probably a photoionization nebula, and to the sharp-line spectrum of

NGC 4151. Thus it is probable, but not certain, that photoionization is the primary input mechanism to the ionized gas in Cyg A.

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