Problem 14:

$$|\psi'\rangle = e^{-\frac{1}{2}\bar{\psi}'\psi'}(|0\rangle - \psi'|1\rangle)$$

$$\langle\psi| = e^{-\frac{1}{2}\bar{\psi}\psi}(\langle 0| + \bar{\psi}\langle 1|)$$

$$\rightsquigarrow \langle\psi|\psi'\rangle = e^{-\frac{1}{2}\bar{\psi}\psi - \frac{1}{2}\bar{\psi}'\psi'}\Big(1 - \bar{\psi}\langle 1|\psi'|1\rangle\Big)$$

$\ominus$

$$= \exp\left\{-\frac{1}{2}\bar{\psi}\psi - \frac{1}{2}\bar{\psi}'\psi'\right\}(1 + \bar{\psi}\psi')$$

$$= \exp\left\{-\frac{1}{2}\bar{\psi}\psi - \frac{1}{2}\bar{\psi}'\psi' + \bar{\psi}\psi'\right\} \quad \#$$

Problem 15:

Recall: $e^{-iH_B t/\hbar}|z'\rangle = e^{-i\omega t a^\dagger a}|z'\rangle = |e^{-i\omega t}z'\rangle$

↳ Lecture time evolution

$$\rightsquigarrow \langle z|e^{-iH_B t/\hbar}|z'\rangle = \exp\left\{-\frac{1}{2}|z|^2 - \frac{1}{2}|z'|^2 + e^{-i\omega t}z^* z'\right\} \quad \#$$

↳ Lecture over-completeness

Recall: $e^{-iH_F t/\hbar}|\psi'\rangle = e^{-i\omega t f^\dagger f}|\psi'\rangle = |e^{-i\omega t}\psi'\rangle$

↳ Lecture time evolution

$$\rightsquigarrow \langle\psi|e^{-iH_F t/\hbar}|\psi'\rangle = \exp\left\{-\frac{1}{2}\bar{\psi}\psi - \frac{1}{2}\bar{\psi}'\psi' + e^{-i\omega t}\bar{\psi}\psi'\right\}$$

↳ Lecture over-completeness

Problem 16:

a) $$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \ddot{x} = \frac{\partial L}{\partial x} = -\omega^2 x \quad \Rightarrow \quad \ddot{x} + \omega^2 x = 0 \qquad (B)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\psi}} = -\frac{i}{2}\dot{\bar{\psi}} = \frac{\partial L}{\partial \psi} = \frac{i}{2}\dot{\bar{\psi}} + \omega\bar{\psi} \quad \Rightarrow \quad \dot{\bar{\psi}} = i\omega\bar{\psi} \qquad (F1)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\bar{\psi}}} = -\frac{i}{2}\dot{\psi} = \frac{\partial L}{\partial \bar{\psi}} = \frac{i}{2}\dot{\psi} - \omega\psi \quad \Rightarrow \quad \dot{\psi} = -i\omega\psi \qquad (F2)$$

b) See Exercise 14 with $V_1(x) = \frac{1}{2}\omega^2 x^2$ and $V_2(x) = \omega = \text{const.}$

$\leadsto$ $\underline{\psi(t) = \psi_0 e^{-i\omega t}}$ and $\underline{\bar{\psi}(t) = \bar{\psi}_0 e^{i\omega t}}$ obvious solution of (F1) and (F2)

Ansatz: $x = x_B + q\,\bar{\psi}_0\psi_0$

Results in $\ddot{x}_B + \omega^2 x_B = 0 \rightarrow \underline{x_B(t) = x_0 \cos\omega t + \frac{\dot{x}_0}{\omega}\sin\omega t}$

Insert into (B) results in $\ddot{q} + \omega^2 q = 0 \quad \Rightarrow$

$$\underline{q(t) = q_0 \cos\omega t + \frac{\dot{q}_0}{\omega}\sin\omega t}$$

In particular for $q_0 = 0 = \dot{q}_0 \Rightarrow q(t) \equiv 0$

$\leadsto$ $x(t) = x_B(t)$ remains reell for all times