5. Homework in "Path Integrals"

Problem 14: Over-completeness of coherent states

Proof the over-completeness of the fermion coherent states,

$$\langle\psi|\psi'\rangle = \exp\left\{-\frac{1}{2}\overline{\psi}\psi - \frac{1}{2}\overline{\psi'}\psi' + \overline{\psi}\psi'\right\} .$$

Hint: ψ and $f^\dagger$ anti-commute and therefore $\psi|1\rangle = -|1\rangle\psi$ but $\psi|0\rangle = |0\rangle\psi$.

Problem 15: Time evolution of boson and fermion coherent states

Let $a, a^\dagger$ and $f, f^\dagger$ be the boson and fermion annihilation and creation operators, respectively. For $H_B := \hbar\omega a^\dagger a$ and $H_F := \hbar\omega f^\dagger f$ show that

$$\langle z|\mathrm{e}^{-\mathrm{i}H_B t/\hbar}|z'\rangle = \exp\left\{-\frac{1}{2}|z|^2 - \frac{1}{2}|z'|^2 + \mathrm{e}^{-\mathrm{i}\omega t} z^* z'\right\} ,$$

$$\langle\psi|\mathrm{e}^{-\mathrm{i}H_F t/\hbar}|\psi'\rangle = \exp\left\{-\frac{1}{2}\overline{\psi}\psi - \frac{1}{2}\overline{\psi'}\psi' + \mathrm{e}^{-\mathrm{i}\omega t}\overline{\psi}\psi'\right\} .$$

Problem 16: The pseudoclassical harmonic oscillator

Consider the pseudoclassical Lagrangian

$$L(\dot{x}, x, \dot{\overline{\psi}}, \overline{\psi}, \dot{\psi}, \psi) = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\omega^2 x^2 + \frac{\mathrm{i}}{2}\left(\overline{\psi}\dot{\psi} - \dot{\overline{\psi}}\psi\right) - \omega\overline{\psi}\psi$$

where x represents a bosonic (even Grassmannn number) and $\overline{\psi}, \psi$ are fermionic (odd Grassmann numbers) degrees of freedom.

a) Derive the pseudoclassical equations of motion for $x, \overline{\psi}$ and ψ.

b) Following the Ansatz $x(t) = x_B(t) + q(t)\overline{\psi}_0\psi_0$ made in Exercise 14, derive the solutions of the equations of motion for initial conditions $x(0) = x_0 + q_0\overline{\psi}_0\psi_0$ and $\dot{x} = \dot{x}_0 + \dot{q}_0\overline{\psi}_0\psi_0$, where $\overline{\psi}(0) = \overline{\psi}_0$, $\psi(0) = \psi_0$.