Statistical decoupling of viewing angle

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1. Contribution

We present a method to statistically decouple the effects of unknown inclination angles on the mass distribution of exoplanets that have been discovered using radial-velocity (RV) techniques. The method can be used in two directions:

1. from the observed mass distribution recover the true-mass distribution, or
2. assume a true-mass distribution and recover a prediction for the observed distribution

2. Background

The planetary mass distribution is a key aspect needed to understand the origin of exoplanets. Currently, RV detections (e.g. [5, 1, 3]) have provided the largest sample of unconstrained systems. However, the RV technique does not provide planet masses directly, but 'minimum' masses, $M_{\rm obs} = M_{\rm T} \sin i$, where i, the line-of-sight inclination angle, and $M_{\rm T}$, the 'true' planet mass, are not known a priori.

3. The problem...

Given an analytical model of the empirical minimum-mass distribution constructed from observation, is it possible to recover the true mass distribution by assuming a random distribution of inclination angles?

4. ...and our concept

Since one is after mass *distributions*, we have developed a formalism that treats the above quantities as *continuous random variables* and works on their *probability density functions* (PDF). Let X, Y, and Z be random variables, such that

- $X = \{M_{\rm obs}\}$ represents the ensemble of observed masses,
- $Y = \{(\sin i)^{-1}\}$ represents the ensemble of correction factors, and
- $Z = XY$ describes an ensemble of corrected masses.

Let also f_X, f_Y, and f_Z be their respective PDF. We wish to obtain f_Z (the distribution of true masses), given that both f_X (the fit to the observed minimum-mass distribution) and f_Y (the distribution of correction factors) are known [4].

References

[1] Butler, R. P. et al. 1996, PASP, 108, 500

[2] Glen, A. G. et al. 2004. "Computing the distribution of the product of two continuous random variables," Computational Statistics & Data Analysis, Elsevier, vol. 44(3), pages 451-464.

[3] Jones, Hugh R. A. et al. 2010, MNRAS, 403, 1703

[4] Lopez, S. & Jenkins, J. S. 2012, submitted

[5] Mayor, M. et al. 1983, A&AS, 54, 495

5. Solution

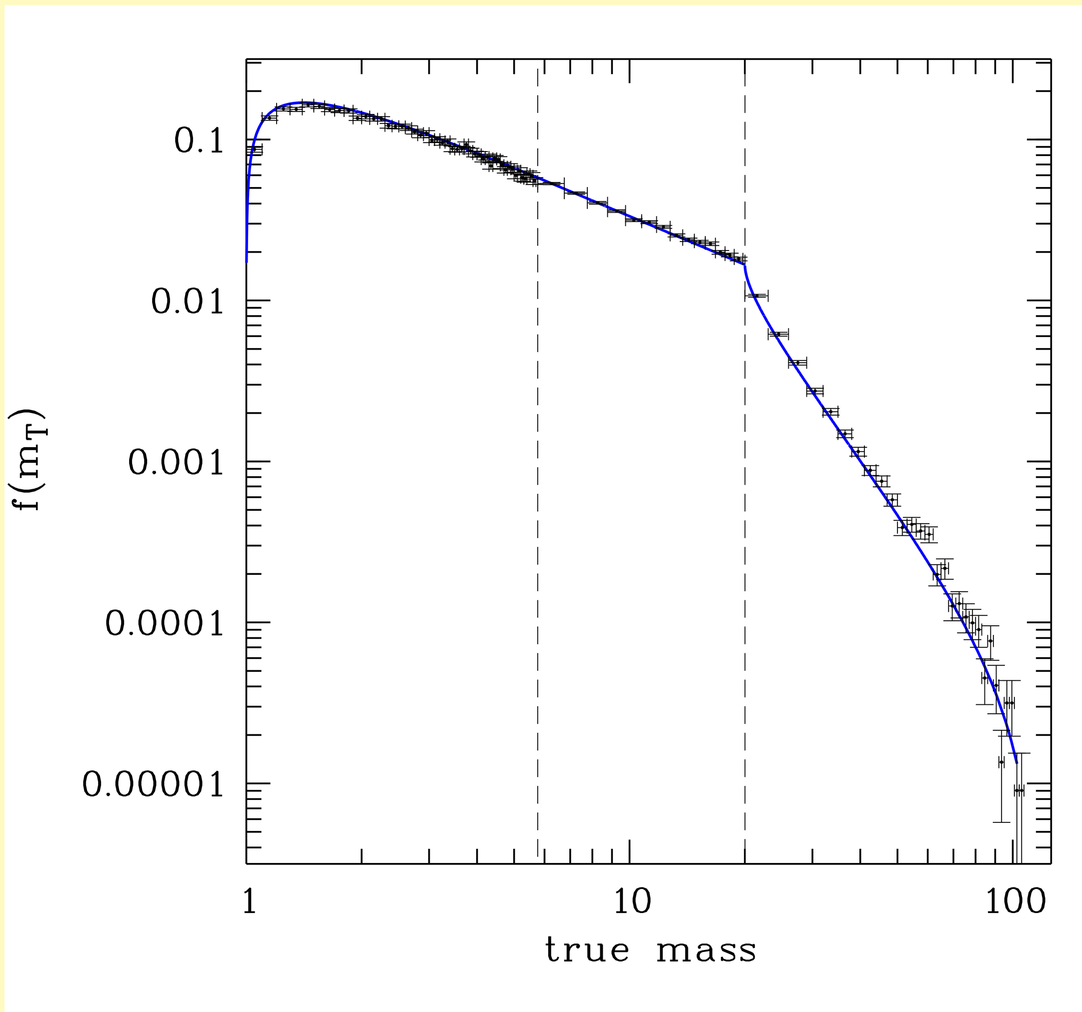
Using the formalism in [2], the PDF of the product of X and Y, $f_Z(z)$, can be expressed as

$$f_Z(z) = \begin{cases} \int_{m_{\rm min}}^{z} f_Y(\frac{z}{u}) f_X(u) \frac{1}{u} du, & m_{\rm min} < z < m_{\rm min}/\sin i_{\rm min}, \\ \int_{z \sin i_{\rm min}}^{z} f_Y(\frac{z}{u}) f_X(u) \frac{1}{u} du, & m_{\rm min}/\sin i_{\rm min} < z < m_{\rm max}, \\ \int_{z \sin i_{\rm min}}^{m_{\rm max}} f_Y(\frac{z}{u}) f_X(u) \frac{1}{u} du, & m_{\rm max} < z < m_{\rm max}/\sin i_{\rm min}, \end{cases} \quad (1)$$

provided that $m_{\rm min}/\sin i_{\rm min} < m_{\rm max}$. This condition is the equivalent of setting a lower limit on i, such that pole-on orbits, producing very large corrections, are excluded from the observed sample.

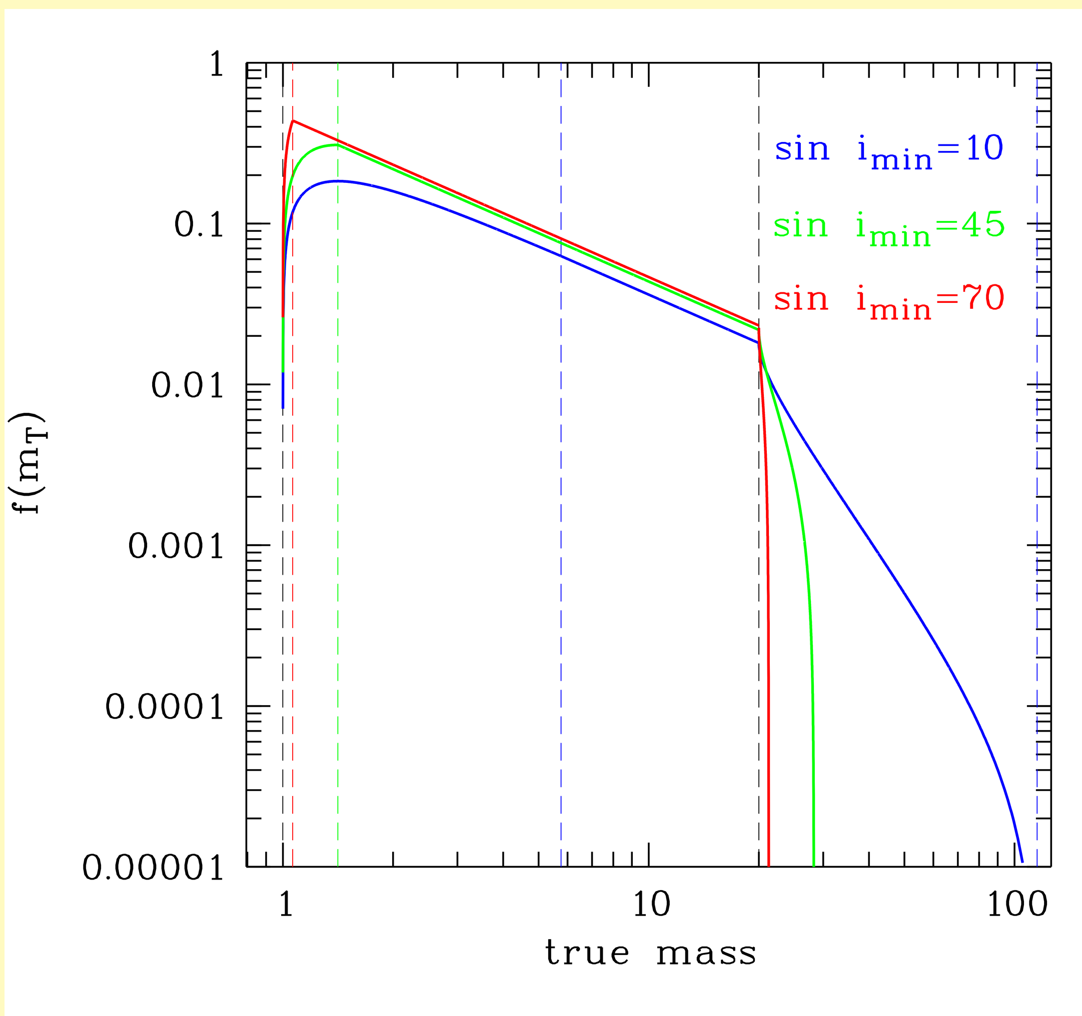
If f_X is a power-law, the solution of Eq.1 is shown [4] to be proportional to ${}_2F_1$, the first hypergeometric function. More complex functions require numerical integration.

6. True-mass recovery



- Data points: simulated distribution of true masses drawn from $f(M_{\rm obs}) \propto M_{\rm obs}^{-1}$ with $m_{\rm min} = 1$ and $m_{\rm max} = 20$, where the individual masses have been corrected randomly for inclination angle.
- Smooth line: true-mass distribution according to Eq. 1 and using $1 < y < (\sin 10^\circ)^{-1}$.
- This kind of models is prone to comparison with current formation/evolution models.

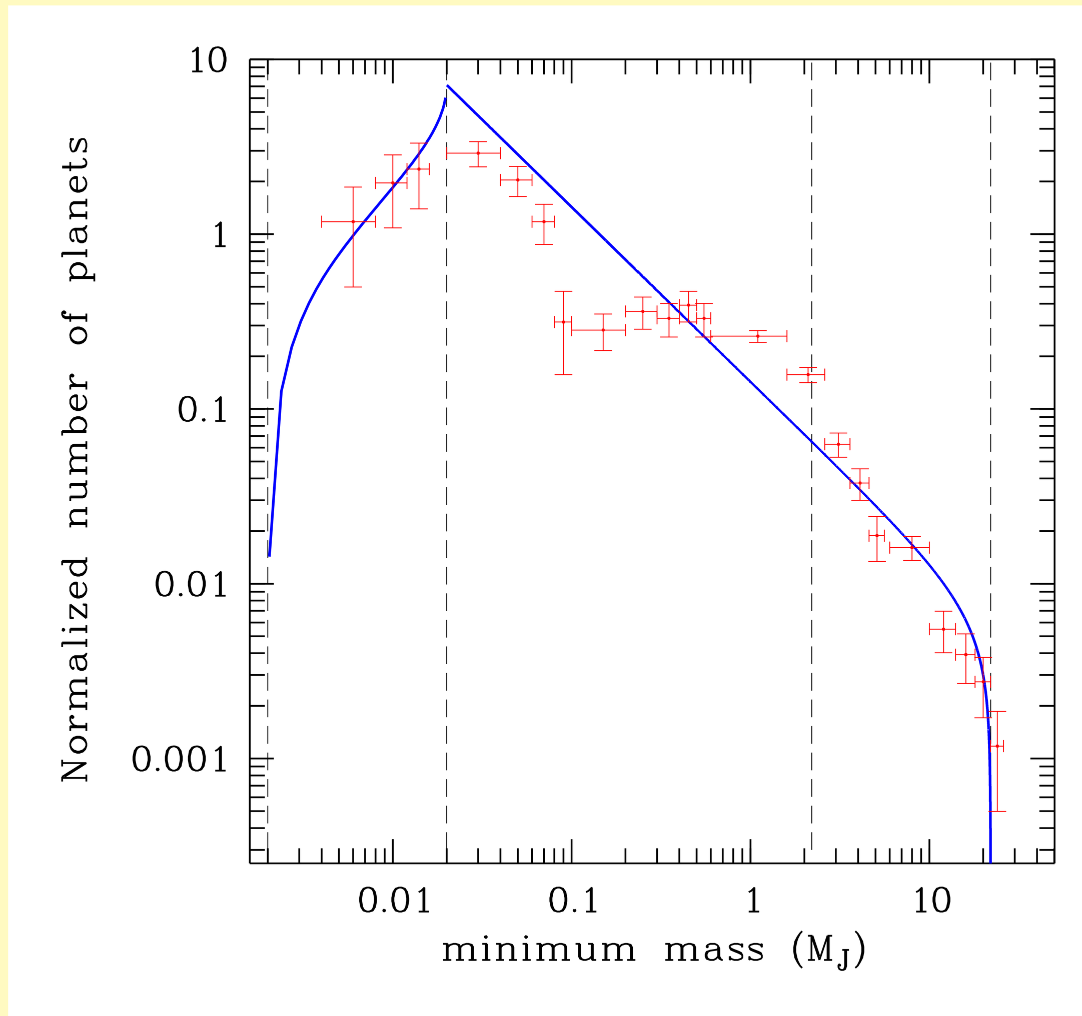
7. Minimum angle



The shape of the predicted distributions is sensitive to the minimum angle considered: near-pole inclinations produce larger corrections, but these are rarer than near face-on inclinations having smaller corrections.

8. Minimum-mass recovery

A more practical application of our method is to compute the expected minimum-mass distribution. In this case we define the random variables $X = \{M_{\rm T}\}$, $Y = \{\sin i\}$, and $Z = XY$, i.e., a prediction for observed masses.



- Data points: observed distribution of minimum masses over a sample of 643 RV discovered exoplanets from http://exoplanet.eu.
- Smooth line: predicted minimum-mass distribution assuming $f(M_{\rm T}) \propto M_{\rm T}^{-1}$, with $m_{\rm min} = 0.02\,{\rm M}_J$, $m_{\rm max} = 22\,{\rm M}_J$, and $\sin 6^\circ < y < 1$.
- Although the power-law part of the predicted curve only poorly fits the data, both of its extremes do seem to better reproduce the data.
- In the low-mass end our prediction provides an alternative explanation for the observed decline, usually explained as due to sample incompleteness.

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