INTERFEROMETRY WEEK at ESO/SANTIAGO
Closure Phases
Markus Wittkowski, ESO Chile

- Why do we want to measure phases ?
  Van Cittert-Zernike theorem. Detection of asymmetric structures. Example of star with spot. Imaging examples by optical speckle interferometry, aperture masking, and long-baseline interferometry.
- Interferometric observables.
  Complex visibility in absence of phase noise. Effect of atmospheric and instrumental phase noise. Solutions: Phase referencing and closure phases.
- Properties of closure phases.
  Closure phases and bispectrum/triple product. Single aperture versus multi-aperture interferometry. Fourier and Bispectum theorems. Number of closure phases; percentage of information.
- Use of closure phases.
  Modelling versus imaging. M: Advantages of triple products and closure phases compared to visibilities. Diameters fitted by closure phases. Example of limb-darkening and starspots. I: Phase recursion methods. Use of additional information. Self calibration and imaging methods. Building block method.

Literature

- John D. Monnier, "An Introduction to Closure Phases", in Principles of Long Baseline Stellar Interferometry, Peter R. Lawson (ed.), Michelson Summer School 1999.
- Andreas Quirrenbach, "Optical Interferometry", Ann. Rev. Astron. Astrophys. 2001, 39 : 353-401
- Oskar von der Lühe, " Interferometrie in der Astronomie", Vorlesungsscript Sommersemester 2001

Phase of the object Fourier transform

- van Cittert-Zernike theorem:

$$V(u,v) = \int\int i(x,y) e^{-2\pi i(ux+vy)} dx dy \tag{1}$$

The visibility modulus $|V|$ can give some information on the object, but does generally not allow to reconstrunct the image.

- Symmetric and asymmetric object intensity distributions

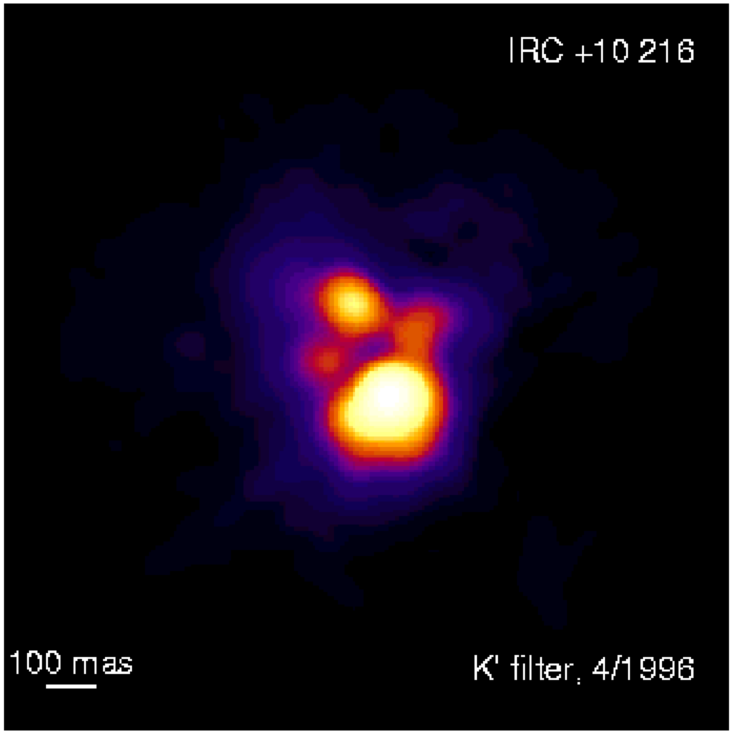
$i(x,y)$ real $\rightarrow V(u,v)$ is hermitian $\rightarrow |V|$ and $|\Phi|$ are point symmetric, Φ changes sign.
$i(x,y)$ is real and point symmetric $\leftrightarrow V(u,v)$ is real, the phase has only values 0 and π.
Phase values different from 0 and π indicate deviations from point symmetry of the object intensity distribution.

- Example of a star with spot

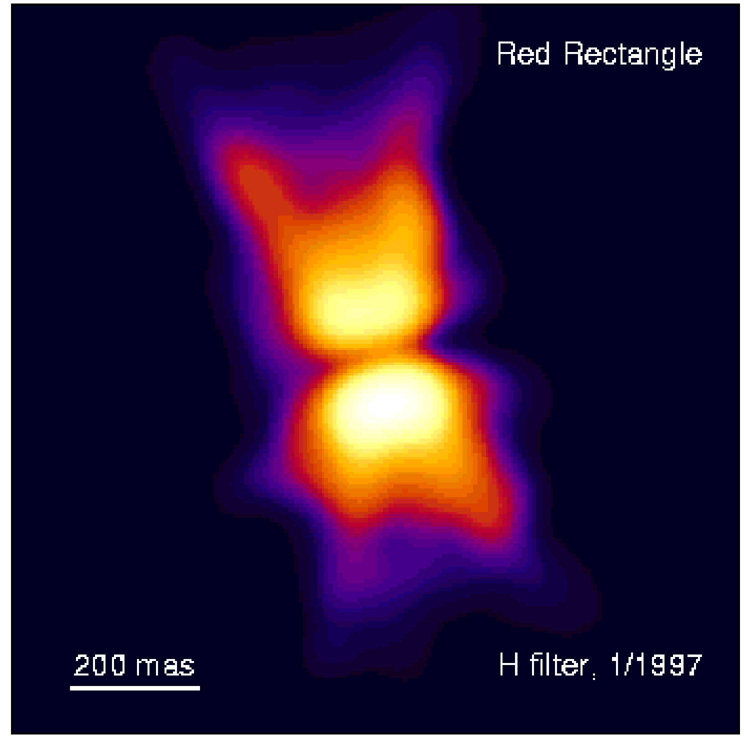
$$\begin{aligned} i(x) &= i_1\delta(x-\Delta/2) + i_2\delta(x+\Delta/2) & (2)\\ |I|^2 &= |I_1|^2 + |I_2|^2 + 2I_1I_2\cos(2\pi u\Delta) & (3)\\ \Phi &= \arctan(\tan(\pi u\Delta)\frac{I_1 - I_2}{I_1 + I_2}) & (4) \end{aligned}$$

$|I|^2$ is invariant for permutations of I_1 and I_2.

Examples from Speckle Interferometry

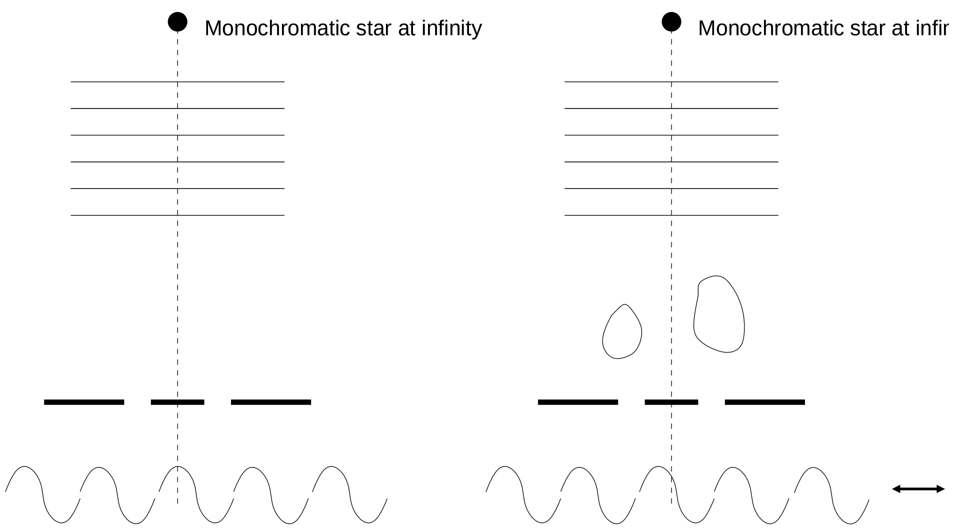


76 mas Speckle-Masking Interferometry of IRC+10216 with the SAO 6m Telescope: Evidence for a clumpy shell structure
G. Weigelt, Y. Balega, T. Blcker, A.J. Fleischer, R. Osterbart and J.M Winters
Astronomy & Astrophysics 333, L51-L54 (1998)



High-resolution bispectrum speckle interferometry and two-dimensional radiative transfer modeling of the Red Rectangle
A.B. Men'shchikov, Y.Y. Balega, R. Osterbart and G. Weigelt
New Astronomy 3, 601-617 (1998)

Effect of atmospheric and instrumental phase noise



- Absence of atmospheric and instrumental phase noise:

  The amplitude and phase of the complex visibility can be directly obtained from the contrast of the fringe pattern and the position (phase) of the white light fringe.

  $\Phi^{1-2}_{\mathrm{meas.}} = \Phi^{1-2}_{\mathrm{object}}$

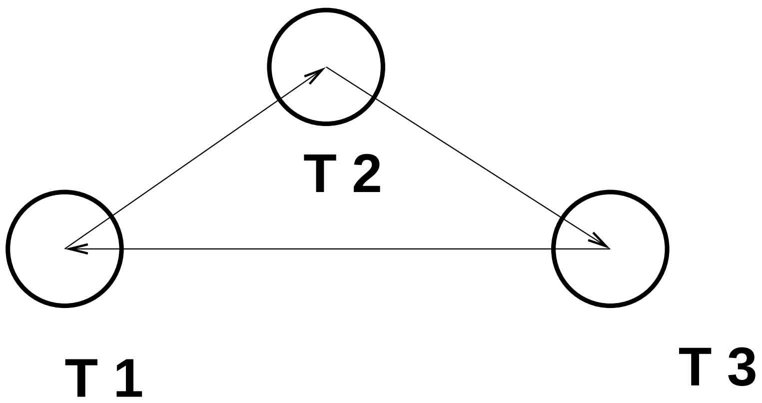
- Phase noise:

  The phase of the fringe pattern, and hence the phase of the complex visibility is corrupted by atmospheric and instrumental phase noise. An incoherent average causes the visibility amplitude to zero.

  $\Phi^{1-2}_{\mathrm{meas.}} = \Phi^{1-2}_{\mathrm{object}} + (\Phi^{1}_{\mathrm{noise}} - \Phi^{2}_{\mathrm{noise}})$

Observables: Squared visibility amplitudes and closure phases.
Object phases can also be retrieved by "phase referencing".

Closure Phase

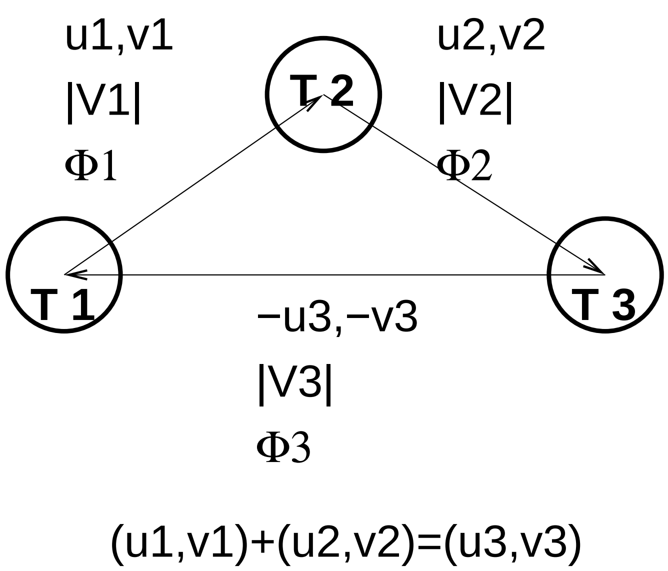


$$\Phi^{123} := \Phi^{1-2} + \Phi^{2-3} + \Phi^{3-1}$$

$$\begin{aligned}\Phi^{123}_{\mathrm{m}} &= \Phi^{1-2}_{\mathrm{o}} + (\Phi^{1}_{\mathrm{n}} - \Phi^{2}_{\mathrm{n}}) + \\ &\quad \Phi^{2-3}_{\mathrm{o}} + (\Phi^{2}_{\mathrm{n}} - \Phi^{3}_{\mathrm{n}}) + \\ &\quad \Phi^{3-1}_{\mathrm{o}} + (\Phi^{3}_{\mathrm{n}} - \Phi^{1}_{\mathrm{n}}) \\ &= \Phi^{123}_{\mathrm{o}}\end{aligned}$$

Dependent closure phases: $\Phi^{123} = \Phi^{12n} + \Phi^{n23} + \Phi^{1n3}$

Properties of the Closure Phase



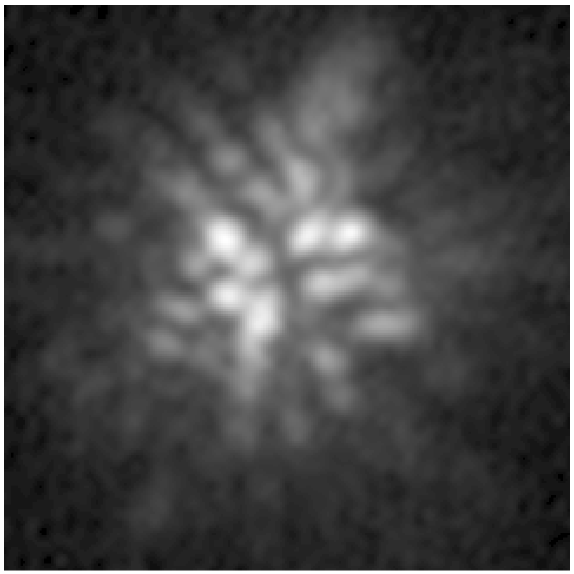
Bispectrum/Triple Product:

$$\begin{aligned} B((u_1,v_1),(u_2,v_2)) &:= V(u_1,v_1)V(u_2,v_2)V^*(u_3,v_3) \\ &= |V_1|e^{i\Phi_1}|V_2|e^{i\Phi_2}|V_3|e^{i\Phi_3} \\ &= |V_1||V_2||V_3|e^{i(\Phi 1+\Phi 2+\Phi 3)} \end{aligned}$$

The phase of the bispectrum or triple product is the closure phase.
In addition to the closure phase, the modulus of the bispectrum (triple amplitude) is a very useful interferometric observable.

- $i(x,y)$ is real $\rightarrow V(u,v)$ is hermitian $\rightarrow V(u,v) = V^*(u,v)$, $V(0,0) = V^*(0,0) \rightarrow \Phi(0,0) = 0$
- $i(x,y)$ is real and pointsymmetric: $i(x,y)$ is hermitian $V(u,v)$ is real $\rightarrow \Phi$ has only values 0 and π.
- Shift theorem: $\hat{F}[g(x-a)] = \hat{F}[g(x)] * e^{-2\pi iua} \rightarrow$ Closure phases are not sensitive to an overall translation of the image $\rightarrow$ Phase of the shortest baseline can be set to zero.

Single aperture versus multi aperture interferometry



One typical speckle interferogram of NGC 1068 taken through a K-band filter at the Russian 6 m telescope. NICMOS 3 array. Exposure time 200 ms. Shown field of view 1.85x1.85 arcsec.

Speckle Interferometry

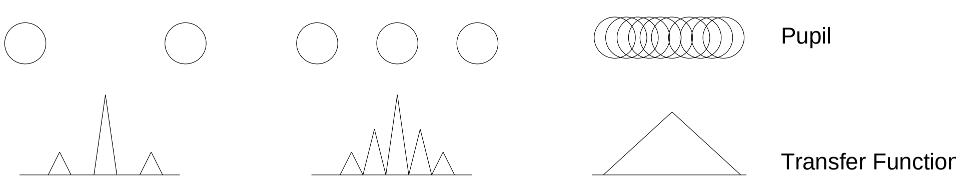
The observation of a source with a single telescope of diameter $D >> r_0$ can be seen as a special case of an interferometer with many subapertures of diameter r_0 where the light interfers in the focal plane of the telescope. Every pair of subapertures produces interference fringes in the image with a random phase. The resulting pattern is called a speckle interferogram.

Each speckle grain has the size of one resolution element λ/D.

The formalism and image processing is basically the same as for long-baseline interferometry.

Labeyrie (1970); Knox, Thompson (1974); Weigelt (1977).

Example: D=6 m, λ=2.2 μm, FOV 7.5 arcsec → image of 100 x 100 resolution elements → corresponds to interferometric array of 100 x 100 apertures.



Scheme (after von der Luehe 2001)

Number of closure phase relations

Number of elements	Number of baselines	Number of closure phases	Number of independent closure phases
N	$\frac{N(N-1)}{2}$	$\frac{N(N-1)(N-2)}{6}$	$\frac{(N-1)(N-2)}{2}$
2	1	0	0
3	3	1	1
4	6	4	3
5	10	10	6
6	15	20	10
50	1225	19600	1176
500	124750	20708500	124251

Percentage of needed information

- A: maximum baseline 100 m, λ=0.5 μm, λ/B=1.0 mas, FOV 1 arcsec → image of 2000 x 2000 px., → 2 million Fourier phases.
- B: maximum baseline 120 m, λ=1.2 μm, λ/B=2.1 mas, FOV 30.9 mas → image of 30 x 30 pixel → 450 Fourier phases.
- C as A, but FOV 10 mas → image of 10 x 10 px. → 50 Fourier phases.

Number of elements N	Number of independent closure phases $\frac{N(N-1)}{2}$	A	B	C
2	0	0 %	0 %	0 %
3	1	5E-5 %	0.2 %	2 %
4	3	0.00015 %	0.7 %	6 %
5	6	0.0003 %	1.3 %	12 %
6	10	0.0005 %	2.2 %	20 %
50	1176	0.06 %	260 %	2E3 %
500	124750	6 %	28E3%	250E3 %

How to use closure phases ?

How can the closure phase (and triple product) information be used to derive astrophysical information ?

- Model fitting
  Parametrized model for the source is needed.
  Best fitting model parameters are determined.
  Triple products and closure phases can directly be used as input.
  Unexpected structures can not be described.
- Imaging
  Full uv coverage:
  Phases can be derived by recursion methods, the final image is obtained by Fourier-re-transform.
  Incomplete uv coverage:
  Good coverage of the uv plane is needed.
  Additional information is used (positivity, limited field of view).
  Less a-priori information is needed as compared to model fitting.
  So far, imaging algorithms require visibility amplitudes and phases.
- Combination of model fitting and imaging
  Model fitting can be used to find a good first guess for the imaging algorithm.

Phase recursion

In case of a full uv coverage (speckle interferometry, LBT), the phases can be retrieved by phase recursion:

$$\begin{aligned} B((u_1,v_1),(u_2,v_2)) &= V(u_1,v_1)V(u_2,v_2)V^*(u_1+u_2,v_1+v_2) \\ \underbrace{\Phi(u_1+u_2,v_1+v_2)}_{\Phi_3} &= \underbrace{\Phi(u_1,v_1)}_{\Phi_1}+\underbrace{\Phi(u_2,v_2)}_{\Phi_2}-\underbrace{\beta((u_1,v_1),(u_2,v_2))}_{\Phi_{123}} \end{aligned}$$

Start values: $\Phi(0,0)=0$, $\Phi(0,1)=0$, $\Phi(1,0)=0$

There are different vector combinations to any given vector $(w_1,w_2)=(u_1+u_2,v_1+v_2)$

For example:
$\Phi(3,2)=\Phi(0,1)+\Phi(3,1)-\beta((0,1),(3,1))$
$\Phi(3,2)=\Phi(0,2)+\Phi(3,0)-\beta((0,2),(3,0))$
$\Phi(3,2)=\Phi(1,1)+\Phi(2,1)-\beta((1,1),(2,1))$ and so on

$\rightarrow$ weighted averaging.

Imaging of incomplete uv data

Input:

- $|V|^2$ ($|V|$), Φ_{123} with incomplete and irregular sampling of the uv-coverage.
- Constraints by positivity, limited field of view (also two or more separated fields).
- Maybe additional model constraints (e.g. point source & nebula).

Deconvolution algorithms require visibility amplitudes and phases !

"Self Calibration" algorithm to use closure phases:

- As the concept of closure phases itself (Jennison 1958), the "self calibration" technique was developed for the radio domain (random phase variations $\gg 1$ rad for VLBI), in the case when phases could not be accurately preserved (e.g. Cornwell & Wilkinson 1981).
- Measured closure phase relations are enforced by modifying some of the phase information of a model image.
- Imaging is then performed in combination with standard deconvolution techniques (CLEAN, MEM, WIPE).

Self calibration

Scheme based on Cornwell & Wilkinson (1981):

1. Choose initial trial image (point source, result of model fit)
2. Calculate visibility amplitudes and phases of trial image at measured uv points.
3. Enforce measured closure phase relations by modifying some of the phases. Use measured visibility amplitudes.
4. Fourier invert modified data and deconvolve the map (using CLEAN, MEM, WIPE...).
5. Use resulting image as next trial image and go to step 2.

Building block method

The "building block method" (Hofmann & Weigelt 1990) is an imaging algorithm that can directly use squared visibility amplitudes and closure phases:

1. Choose an image size ($n \times n$ pixel) and start with all values set to zero.
   Set one (the central) pixel to one.
2. Try all n^2 possibilities to increase one of the pixels by one.
   For each possibility, calculate the squared visibility amplitudes and closure phases of the new image at the data points.
3. Choose the possibility with the lowest least square distance to the measured data.
4. Go to 2.