Introduction to millimeter Interferometry

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Bibliography:

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Why Millimeter ?

- This is the next best atmospheric window longward of the IR cutoff.
- Most of the Universe is cold (today).
- Emission from a black body at 10 K peaks in the mm domain.
- The most abundant molecules (e.g. CO, HCN) have their fundamental rotation lines in the mm domain.(*)
  Exceptions: the lighter hydrides have their fundamental at higher frequencies, and more complex, heavier, molecules have their fundamental at longer wavelengths)
- The low temperature also implies the peak of the emission from molecules (even heavy ones) is in the mm domain.

Why Interferometry ?

- Angular resolution goes (only) as λ/D
- To get $1''$ at 1 mm, we need D $=$ 200 m
- As far as I know, it is somewhat difficult to build a fully steerable football stadium
- Specially if it has to be (blindly) oriented within $0.5''$ precision
- So we have to rely on the Young's holes...

Basic principle

- The antenna produces an output Voltage proportional to the linear superposition of the incident electric field pattern. For a simple monochromatic case:

$$U(t) = E\cos(2\pi\nu t + \phi) \tag{1}$$

- In the receiver, a `mixer` superimposes the field generated by a `local oscillator` to the antenna output.

$$U_{LO}(t) = Q\cos(2\pi\nu_{LO}t + \phi_{LO}) \tag{2}$$

- The `mixer` is a non-linear element (diode) whose output is

$$I(t) = a_0 + a_1(U(t) + U_{LO}(t)) + a_2(U(t) + U_{LO}(t))^2 + a_3(U(t) + U_{LO}(t))^3 + ... \tag{3}$$

The second order (quadratic) term of Eq.3 can be expressed as

$$\begin{aligned} I(t) &= a_2E^2\cos^2(2\pi\nu t + \phi) + 2a_2EQ\cos(2\pi\nu t + \phi)\cos(2\pi\nu_{LO}t + \phi_{LO}) \\ &+ a_2Q^2\cos^2(2\pi\nu_{LO}t + \phi_{LO}) \end{aligned} \tag{4}$$

- Developing the product of the two cosine functions, we obtain

$$I(t) = ... + a_2 EQ \cos(2\pi(\nu - \nu_{LO})t + \phi - \phi_{LO}) + ... \tag{5}$$

There are obviously other terms in $\nu + \nu_{LO}$, $2\nu_{LO}$, 2ν, $3\nu_{LO} \pm \nu$, etc... in the above equation, as well as terms at very different frequencies like ν, 3ν, etc...

- By inserting a filter at the output of the `mixer`, we can select only the term such that

$$\nu_{IF} - \Delta\nu/2 \leq |\nu - \nu_{LO}| \leq \nu_{IF} + \Delta\nu/2 \tag{6}$$

where ν_{IF}, the so-called *Intermediate Frequency*, is a frequency which is significantly different from the original signal frequency ν (which is often called the *Radio Frequency* ν_{RF}).

- Hence, after mixing and filtering, the output of the `receiver` is

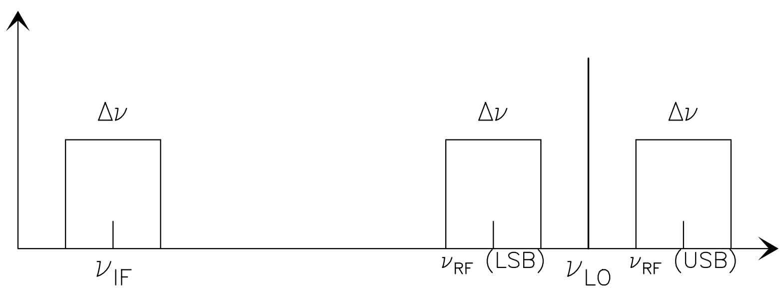
$$I(t) \propto EQ \cos(\pm(2\pi(\nu - \nu_{LO})t + \phi - \phi_{LO})) \tag{7}$$

- The receiver output is

$$I(t) \propto EQ\cos(\pm(2\pi(\nu - \nu_{LO})t + \phi - \phi_{LO})) \quad (8)$$

-
  - changed in frequency: $\nu \longrightarrow \nu - \nu_{LO}$ or $\nu \longrightarrow \nu_{LO} - \nu$
  - proportional to the original electric field of the incident wave: $\propto E$
  - with a phase relation with this electric field:
    $\phi \longrightarrow \phi - \phi_{LO}$ or $\phi \longrightarrow \phi_{LO} - \phi$
  - proportional to the `local oscillator` voltage: $\propto Q$

- The frequency change, usually towards a lower frequency, allows to select ν_{IF} such that amplifiers and transport elements are easily available for further processing.

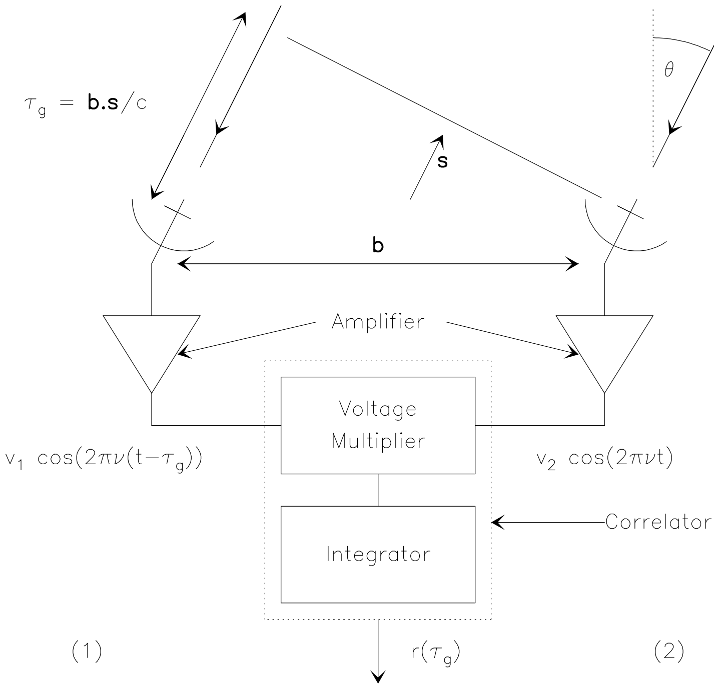


Relation between the IF, RF and local oscillator frequencies in an heterodyne system

- The mixer described before accepts simultaneously frequencies which are
  - higher than the `local oscillator` frequency: this is called Upper Side Band (USB) reception
  - lower than the `local oscillator` frequency: this is called Lower Side Band (LSB) reception

  and cannot *a priori* distinguish between them. This is called Double Side Band (DSB) reception.
- Some receivers are actually insensitive to one of the frequency range, either because a filter has been placed at the receiver input, or because their response is very strongly frequency dependent. Such receivers are called Single Side Band (SSB) receivers.
- An important property of the receiving system expressed by Eq.?? is that the sign of the phase is changed for LSB conversion.
- This property can be easily retrieved recognizing that the **Frequency** ν is the time derivative of the **Phase** ϕ.

The Heterodyne Interferometer



Schematic diagram of the two-antenna radio interferometer. $\tau_g = b.s/c$ is called the geometrical delay

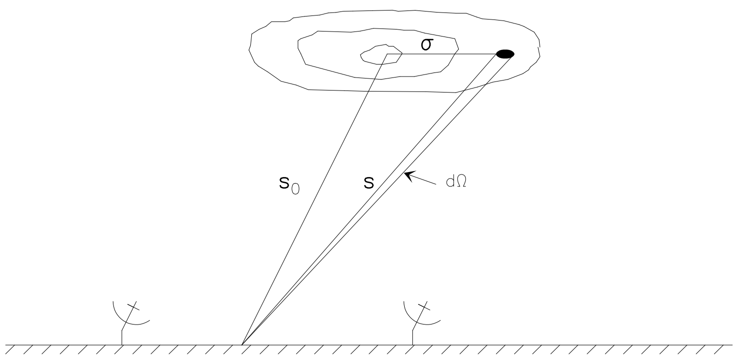
Let us forget the frequency conversion for some time, i.e. assume $\nu_{IF} = \nu_{RF}$...

- The input (amplified) signals from 2 elements of the interferometer are processed by a `correlator`, which is just a voltage multiplier followed by a time integrator.
- With one incident plane wave, the output $r(t)$ is

$$r(t) =< v_1 \cos(2\pi\nu(t - \tau_g(t)))v_2 \cos(2\pi\nu t) >= v_1 v_2 \cos(2\pi\nu\tau_g(t)) \quad (9)$$

  where τ_g is the geometrical delay $\tau_g(t) = (\mathbf{b}.\mathbf{s})/c$

- As τ_g varies slowly because of Earth rotation, $r(t)$ oscillates as a cosine function, and is thus called the **fringe pattern**. As we had shown before that v_1 and v_2 were proportional to the electric field of the wave, the correlator output (fringe pattern) is thus proportional to the power of the wave.

- 

Position vectors used for the expression of the interferometer response to an extended source, schematically represented by the iso-contours of the sky brightness distribution.

Source Size Effects

- The signal power received from a sky area $d\Omega$ in direction $\mathbf{s}$ is (see Fig. for notations)

$$A(\mathbf{s})I(\mathbf{s})d\Omega d\nu$$

over bandwidth $d\nu$, where $A(\mathbf{s})$ is the antenna power pattern (assumed identical for both elements, more precisely $A(\mathbf{s}) = A_i(\mathbf{s})A_j(\mathbf{s})$ with A_i the voltage pattern of antenna i), and $I(\mathbf{s})$ is the sky brightness distribution

$$dr = A(\mathbf{s})I(\mathbf{s})d\Omega d\nu \cos(2\pi\nu\tau_g) \quad (10)$$

$$r = d\nu \int_{Sky} A(\mathbf{s})I(\mathbf{s}) \cos(2\pi\nu\mathbf{b}.\mathbf{s}/c)d\Omega \quad (11)$$

- Two implicit assumptions have been made in deriving Eq.11.
  - We assumed incident plane waves, which implies that the source must be in the far field of the interferometer.
  - We used a linear superposition of the fringes from the incident waves, which implies that the source must be spatially incoherent.
- These assumptions are quite valid for most astronomical sources, but may be violated under special circumstances (e.g. VLBI observations of solar system objects would violate the first assumption, or masers for the second one.)

- When the interferometer is tracking a source in direction $\mathbf{s}_o$, with $\mathbf{s} = \mathbf{s}_o + \sigma$

$$
\begin{aligned}
r &= d\nu \cos(2\pi\nu\frac{\mathbf{b}.\mathbf{s}_o}{c}) \int_{Sky} A(\sigma)I(\sigma)\cos(2\pi\nu\mathbf{b}.\sigma/c)d\Omega \\
&- d\nu \sin(2\pi\nu\frac{\mathbf{b}.\mathbf{s}_o}{c}) \int_{Sky} A(\sigma)I(\sigma)\sin(2\pi\nu\mathbf{b}.\sigma/c)d\Omega
\end{aligned}
\tag{12}
$$

- We define the *Complex Visibility*

$$
V = |V|e^{i\phi_V} = \int_{Sky} A(\sigma)I(\sigma)e^{(-2i\pi\nu\mathbf{b}.\sigma/c)}d\Omega \tag{13}
$$

  which resembles a Fourier Transform...

- We thus have

$$
\begin{aligned}
r &= d\nu\ \cos(2\pi\nu\frac{\mathbf{b}.\mathbf{s}_o}{c})|V|\cos(\phi_V) - d\nu\ \sin(2\pi\nu\frac{\mathbf{b}.\mathbf{s}_o}{c})|V|\sin(\phi_V) \\
&= d\nu\ |V|\cos(2\pi\nu\tau_g - \phi_V)
\end{aligned}
\tag{14}
$$

  i.e. the correlator output is proportional to the amplitude of the visibility, and contains a phase relation with the visibility.

Finite Bandwidth

- Integrating over $d\nu$,

$$R = \frac{1}{\Delta\nu} \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} |V| \cos(2\pi\nu\tau_g - \phi_V) d\nu \tag{15}$$

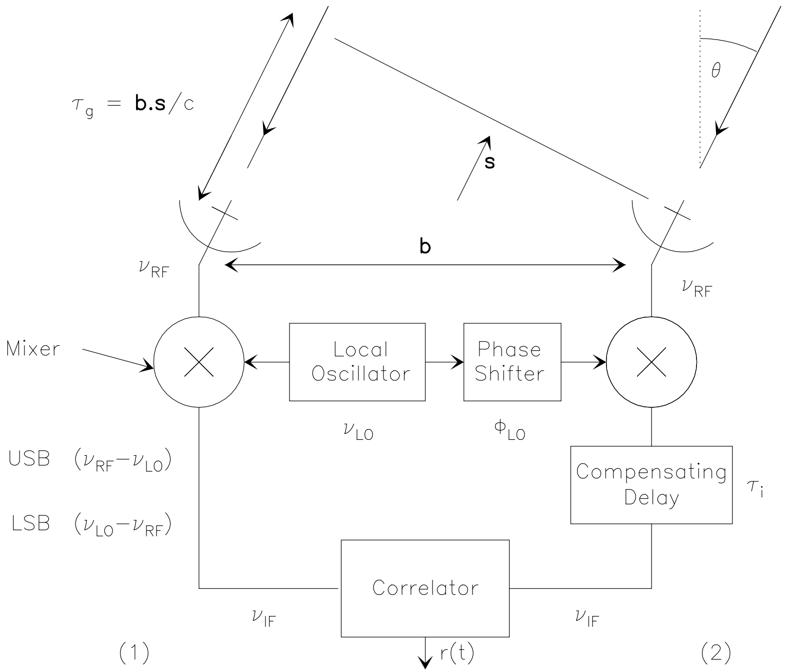
- Using $\nu = \nu_0 + n$

$$R = \frac{1}{\Delta\nu} \int_{-\Delta\nu/2}^{\Delta\nu/2} |V| \cos(2\pi\nu_0\tau_g - \phi_V + 2\pi n\tau_g) dn \tag{16}$$

$$= |V| \cos(2\pi\nu_0\tau_g - \phi_V) \frac{\sin(\pi\Delta\nu\tau_g)}{\pi\Delta\nu\tau_g} \tag{17}$$

- The fringe visibility is attenuated by a $\sin(x)/x$ envelope, called the bandwidth pattern, which falls off rapidly. 1% loss in visibility $<==>$ $|\Delta\nu\tau_g| \simeq 0.078$, or with $\Delta\nu = 500$MHz and a baseline length $b = 100$m, when the zenith angle is 2 arcmin only.
- The ability to track a source for a significant hour angle coverage requires proper compensation of the geometrical delay when a finite bandwidth is desired.

Delay Tracking and Frequency Conversion

- Delay lines with mirrors (as in optics...) are impractical: avoiding diffraction losses requires $D_m^2 \gg B\lambda$, i.e. $D_m \gg 9$ m for ALMA ($\lambda = 7$ mm, $B = 12$ km).
- The compensating delay must be introduced after one (or several) frequency conversion(s)
- 

2-element heterodyne interferometer with delay tracking after frequency conversion

- For USB conversion, the phase changes of the input signals before reaching the correlator are

$$\phi_1 = 2\pi\nu\tau_g = 2\pi(\nu_{LO} + \nu_{IF})\tau_g \tag{18}$$
$$\phi_2 = 2\pi\nu_{IF}\tau_i + \phi_{LO} \tag{19}$$

since the delay is inserted at frequency ν_{IF}

- $\Delta\tau = \tau_g - \tau_i$ being the delay tracking error, the correlator output is

$$\begin{aligned} r &= |V| \cos(\phi_1 - \phi_2 - \phi_V) \\ r_{\rm USB} &= |V| \cos(2\pi(\nu_{LO}\tau_g + \nu_{IF}\Delta\tau) - \phi_V - \phi_{LO}) \\ r_{\rm LSB} &= |V| \cos(2\pi(\nu_{LO}\tau_g - \nu_{IF}\Delta\tau) - \phi_V - \phi_{LO}) \end{aligned} \tag{20}$$

- When the two sidebands are superposed,

$$r_{\rm DSB} = 2|V| \cos(2\pi(\nu_{LO}\tau_g - \phi_V - \phi_{LO})) \cos(2\pi\nu_{IF}\Delta\tau) \tag{21}$$

i.e. the amplitude is modulated by the delay tracking error.

- We use sideband separation to avoid this problem. The delay tracking error should then be kept small compared to the bandwidth $\Delta\tau \ll 1/\Delta\nu$.

Fringe Stopping

- With the Earth rotation, the cosine term of Eq.17 modulates the correlator output with a *natural fringe rate* of

$$\nu_{LO}\frac{d\tau_g}{dt} \simeq \Omega_{earth}\frac{\mathrm{b}\nu_{LO}}{c} \tag{22}$$

which is of order of 10 Hz for b$= 300$ m baselines and $\nu_{LO} = 100$ GHz, or 2$''$ angular resolution (since the fringe rate only depends on the effective angular resolution).

- Fringe rate too high for digital sampling of the visibility. Exception: VLBI, although resolutions are < 1 mas.
- Usual technique: modulate the phase of the `local oscillator` ϕ_{LO} such that

$$\phi_{LO}(t) = 2\pi\nu_{LO}\tau_g(t)$$

at any given time. Then is a slowly varying output, which would be constant for a point source at the reference position (also called the *delay tracking center*). This process is called **Fringe Stopping**.

Complex Correlator

- After fringe stopping
$$r_r = A_o|V|\cos(\pm 2\pi\nu_{IF}\Delta\tau - \phi_V) \tag{23}$$
we can no longer measure the amplitude $|V|$ and the phase ϕ_V separately.
- A second correlator, with one signal phase shifted by $\pi/2$ becomes necessary. Its output is
$$r_i = A_o|V|\sin(\pm 2\pi\nu_{IF}\Delta\tau - \phi_V) \tag{24}$$
- With both correlators, we measure directly the real r_r and imaginary r_i parts of the **complex visibility** r. The device is thus called a **complex correlator**.
- **Note:** A delay tracking error $\Delta\tau$ appears as a phase slope as a function of frequency, with
$$\phi(\nu_{IF}) = \pm 2\pi\nu_{IF}\Delta\tau \tag{25}$$
with the $+$ sign for USB conversion, and the – sign for LSB conversion.

Spectroscopy

- Observing spectral lines requires to make synthesis images at a large number of closely spaced frequencies
- This can be done by implementing a large number of multipliers to calculate the correlation function as a function of lag τ. Noting that each lag results in a phase shift $\tau\nu$ at frequency ν, $V(u,v,\tau)$ is given by:

$$V_\nu(u,v,\tau) = \int V(u,v,\nu).e^{i2\pi\tau\nu}.d\nu \tag{26}$$

  This Fourier transform can be inverted to retrieve the complex visibility $V(u,v,\nu)$
- This is feasible because the complex visibility $V(u,v,\nu)$ of the source varies on timescales (dominated by Earth rotation) which is significantly larger than the lag step $\tau = 1/\Delta\nu$.
- This additional lag machinery allows coarse compensation of the geometric delay. Compensation of the fine delay $\Delta\tau = \mathrm{mod}(\tau_g, 1/\Delta\nu)$ can be done by a simple phase slope as function of frequency.
- For n lags, the spectral channel width is $\Delta\nu/n$ and the low loss delay tracking condition $\delta\nu\Delta\tau \ll 1$ is satisfied if $n \gg 1$.

In summary...

In a **heterodyne interferometer**

1. **Sign of Phase** depends on sideband conversion
2. **Frequency** is the time derivative of the **Phase**
3. **Finite Bandwidth** implies *delay tracking*
4. a **Delay error** produces a linear phase slope as function of frequency
5. **Fringe Stopping** makes life easier, but implies a *Complex Correlator*
6. **High resolution spectroscopy** is natural
7. **Delays** can be implemented digitally
8. a **Complex Correlator** directly measures the **Visibility**

Instrumental Effects

- Delay tracking and fringe stopping require good knowledge of the geometrical delay, i.e. of the baseline coordinates.
- Accurate phase lock systems are required to control ϕ_{LO}
- Antenna deformations (e.g. thermal expansion) may change the geometrical delay. Proper control of the antenna focus is required.
- The atmosphere may introduce a variable, random **delay** above each antenna. Note that this is a delay, although it is often referred to as a phase.
- and of course, there is noise...

Calibration: Why ?

- ϕ_{LO} may vary with time $\longrightarrow$ Phase Calibration
- The LO power may vary with time $\longrightarrow$ Amplitude Calibration
- The atmospheric transparency may vary with time $\longrightarrow$ Amplitude Calibration
- The atmosphere introduces a non-geometric delay, because of varying water vapor $\longrightarrow$ Phase Calibration
- The geometric delay must be known $\longrightarrow$ Baseline Calibration
- The instrumental delay must be known $\longrightarrow$ Delay Calibration
- The amplitude and phase response as $f(\nu)$ must be known $\longrightarrow$ Bandpass Calibration

Calibration: How ?

- **Observe sources for which you know the answer (i.e. the visibility)**
- `Baseline`: on an ensemble of point sources of known direction (since $\phi = \mathbf{b}.\mathbf{s}/c$, you can determine $\mathbf{b}$ if you have an ensemble of $\mathbf{s}$).
- `Bandpass and delay`: on a strong continuum point source
- `Atmospheric transparency`: from atmospheric models
- `Amplitude`: observe point source of known flux
- `LO phase`: observe point sources regularly
- `Atmospheric phase`: **sorry, you cannot...**

Atmospheric Phase

- Can not (usually) be calibrated because
  - Time scale is short (typically 100 sec)
  - Antennas cannot re-point so quickly
- However, atmospheric phases are usually small enough, i.e. much less than 1 radian.
- More precisely
  - the phase error is $\propto \nu$, because the atmosphere actually introduces a varying **pathlength** $\delta\mathrm{Path}$
  - the pathlength fluctuations goes as $D^{0.8-0.3}$
  - up to an outer scale of $D \approx 1000 - 2000$ m
  - the timescale is simply D/v_{wind} (frozen atmosphere model), typically 30 seconds for 300 m.
  - $\delta\mathrm{Path}(300\ \mathrm{m}) \lessapprox 200 - 300\mu$m, $\ll \lambda$.
  - this result in an the atmospheric seeing of about 0.3 – 1″, nearly frequency independent.
- Could be predicted by Water Vapor Monitoring, because water is (by large) the dominant refractive component of the atmosphere.

The notion of Noise Temperature, Brightness and Flux

- The output power of any electronic device is simply given by its **noise temperature**. This is the **Nyquist's relation ($W = k.T.\Delta\nu$)**.
- An antenna is an electronic device. Its impedance can be derived from its optical characteristics. We define its equivalent noise temperature $T_{\rm sys}$ as the physical temperature of a resistor with same impedance delivering the same output power.
- The power received from a source of flux density S_ν (Watt.m^2.Hz^{-1}) by an antenna of collecting area A (m^2), aperture efficiency $\eta_{\rm A}$ over bandwidth $\Delta\nu$ (Hz) is

$$P_s = \frac{1}{2}\eta_{\rm A} A \Delta\nu S_\nu = k T_A \Delta\nu \tag{27}$$

- T_A is the *antenna temperature* of the source

$$S_\nu = \frac{2k}{\eta_{\rm A} A} T_A = \mathcal{J} T_A$$

- In an integration time t, there are about $2\Delta\nu$ independent samples, giving a signal to noise:

$$S/N = C \frac{T_A}{T_{\rm sys}} \frac{1}{\sqrt{\Delta\nu t}} \tag{28}$$

where C is a constant of order 1, depending on the details of receiving system.

- (Rayleigh-Jeans) brightness temperature T_b and Flux density S_ν are related by the simple integral
$$S_\nu = 2kT_\mathrm{b}\Omega_b/\lambda^2 \tag{29}$$
Ω_b being the solid angle and λ the wavelength (k the Boltzmann constant)
- and the antenna primary beam and aperture are related by the antenna equation
$$\Omega_\mathrm{P} A_\mathrm{eff} = \lambda^2 \tag{30}$$
T_A and T_b are similar quantities: $T_A = T_\mathrm{b}$ for sources filling the antenna beam.
- **Noise temperatures** when referred to the aperture plane of the antenna are directly comparable to the (Rayleigh-Jeans) brightness temperatures of the sky.
- This allows a very simple and direct comparison between the sky emission and the total system noise. The noise equation is
$$\Delta T_\mathrm{b} = \frac{T_\mathrm{sys}}{\sqrt{\Delta\nu t}} \tag{31}$$
where $\Delta\nu$ is the bandwidth, t the integration time, T_sys the so-called **system temperature** and ΔT_b the resulting noise.

Sensitivity of an interferometer

- For a single baseline interferometer, the noise level is

$$\sigma = \frac{\sqrt{2}kT_{\rm sys}}{\eta_{\rm A}\eta_{\rm Q}A\sqrt{\Delta\nu t}} = \frac{1}{\sqrt{2}}\frac{\mathcal{J}T_{\rm sys}}{\eta_{\rm Q}\sqrt{\Delta\nu t}} \tag{32}$$

  where A is the collecting area of **one** antenna, and $\eta_{\rm Q}$ is the correlator efficiency (between 0.64 and 1, for (good) digital correlators). $\mathcal{J}$ is the Jy/K conversion factor for **one** antenna.

- σ is $\sqrt{2}$ worse than for an antenna of equal *total* collecting area, but no reference is required, so t can be $2\times$ longer...
- This *"loss"* of sensitivity compared to a single-dish is due to the lack of analysis of the auto-correlation products.

- With n antennas, there are $n(n-1)/2$ independent measurements, so

$$\sigma = \frac{2kT_{\mathrm{sys}}}{\eta_{\mathrm{A}}\eta_{\mathrm{Q}}\eta_{\phi}A\sqrt{n(n-1)\Delta\nu t}} = \frac{\mathcal{J}T_{\mathrm{sys}}}{\eta_{\mathrm{Q}}\eta_{\phi}\sqrt{n(n-1)\Delta\nu t}} \tag{33}$$

where the term η_{ϕ} is the decorrelation factor due to phase noise:

$$\eta_{\phi} = e^{-(\Delta\phi)^2/2} \tag{34}$$

with $\Delta\phi$ the baseline-based phase noise.

- As $A_{\mathrm{eff}} = \eta_{\mathrm{A}}A = \lambda^2/\Omega_{\mathrm{P}}$ and $S_{\nu} = 2kT_{\mathrm{b}}\Omega_{\mathrm{S}}/\lambda^2$, Eq.33 simplifies too

$$\Delta T_{\mathrm{b}} = \frac{T_{\mathrm{sys}}}{n\sqrt{\Delta\nu t}}\left(\frac{\theta_{\mathrm{P}}}{\theta_{\mathrm{S}}}\right)^2 \tag{35}$$

i.e. that of a fictitious antenna with the same **total** collecting area (nA_{eff}, same primary beam θ_{P}, corrected for the **beam dilution factor** $(\theta_{\mathrm{P}}/\theta_{\mathrm{S}})^2$, except for small losses η_{Q} and η_{ϕ}

Calibration of single-dish mm telescope

- So far, we had neglected atmospheric opacity...
- **The noise power detected by the receiver is** $W_A + W_{rec}$, the sum of the power received by the antenna and the noise generated by the receiver + transmission lines.
- **The power received by the antenna is** $W_A = \eta W_{\rm sky} + (1-\eta) W_{\rm ground}$, where η is the *forward efficiency* (coupling to the sky)
- **Using Nyquist's relation** $(W = k.T.\Delta\nu)$ we can convert that to temperatures

$$T_{\rm ant} = T_{\rm rec} + \eta T_{\rm sky} + (1-\eta) T_{\rm ground} \quad (36)$$

$$T_{\rm sky} = e^{-\tau} T_{\rm bgg} + (1 - e^{-\tau}) T_{\rm atm} + e^{-\tau} T_A^* \quad (37)$$

  where τ is the atmospheric opacity.
- T_A^* is the antenna temperature of the astronomical source. It is equal to the brightness temperature of the source if the source fills the primary beam.
- Knowledge of $T_{\rm rec}$ and η and measurement of the output power allows determination of the atmospheric opacity τ when a reasonable model for $T_{\rm atm}$ can be used.

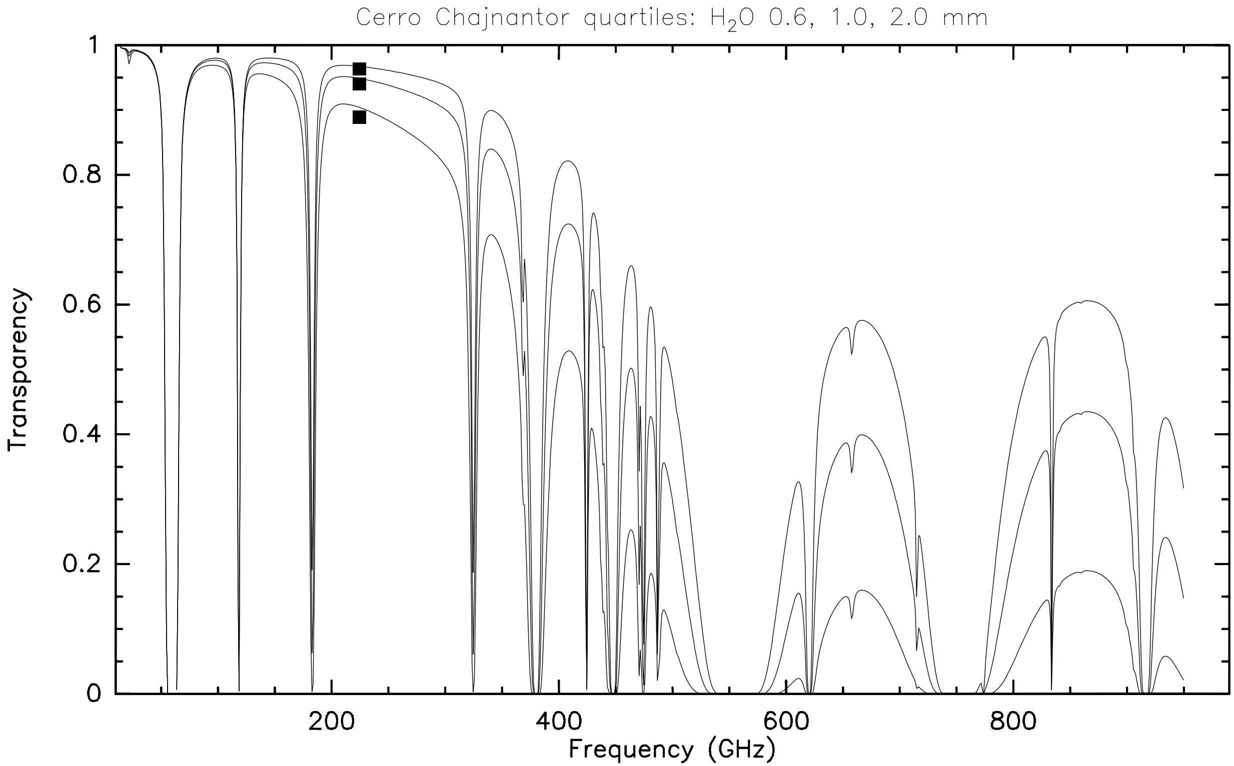
Antenna and System Temperatures

- The noise temperature $T_{\rm ant}$ refers to the aperture plane of the antenna. If we instead refer to the brightness temperature scale of the source, we can express the **system temperature**

$$T_{\rm sys} = \frac{e^{\tau}}{\eta}(T_{\rm rec} + \eta T_{\rm sky} + (1-\eta)T_{\rm ground}) \qquad (38)$$

- The **System Temperature** T_{sys} is the equivalent noise temperature of all the instrumental system including the atmosphere emission. It is directly comparable to the source brightness temperature.
- Best values of T_{sys} for mm radio telescopes ($e.g$ IRAM 30-m or PdBI) are:
  - $T_{\rm sys}(\text{3mm or 90 GHz}) \simeq 60 - 100K$
  - $T_{\rm sys}(\text{1.3mm or 230 GHz}) \simeq 250 - 350K$
- Typical values of T_A^* observed with single-dish 15-m antennas (PdBI) are:
  - T_A^* (CO J=1-0 in dark cloud) $\simeq 5 - 10$ K
  - T_A^* (CO J=2-1 in Keplerian disk) $\simeq 0.5$ K
  - T_A^* (CO in high-z galaxy) $\simeq 5$ mK
- **Ratio:** $T_A^*/T_{\rm sys} \sim 10^{-2} - 10^{-6}$

Atmospheric transparency at mm and submm wavelengths.



Sensitivity of current mm arrays

- The antenna Jy/K gain is $J = 2k/(A_{\rm eff}) \simeq 35 \;\; {\rm Jy/K}$, at 1.3mm for the ALMA antennas, 30 for the IRAM antennas

$$\Delta S_\nu = \frac{J \cdot T_{\rm sys}}{\eta_Q \sqrt{2 N_{\rm base} \Delta\nu t_{\rm int}}} \qquad (39)$$

  - $T_{\rm sys} = 300$ K, $N_{\rm base} = n(n-1)/2 = 10$, $\eta_{\rm Q} = 0.6$ (correlator loss & decorrelation factor)

- **In continuum,** $\Delta\nu = 500$ MHz
  - 1 hour integration time $\longrightarrow$ 2.8 mJy/beam
  - Best: 24 hour integration time $\longrightarrow$ 0.5 mJy/beam

- **For spectral lines** of line-width $\Delta\nu = 100$ kHz (or ΔV = 0.1 km/s)
  - 10 hours integration time $\longrightarrow$ 60 mJy/beam
  - therefore 1.4 K at 1$''$ of angular resolution at 230 GHz (140 K at $0.1''$...)

- ALMA will be 30 to 100 times more sensitive (2016 baselines instead of 10, better $T_{\rm sys}$...)

Orders of magnitude

$$\begin{aligned}
\Delta\nu &= 500\ \mathrm{MHz} \\
t &= 6\ \mathrm{h} \\
(\Delta\nu t)^{-\frac{1}{2}} &= 3\,10^{-7} \\
A &= 1000\ \mathrm{m}^2 \\
\eta_{\mathrm{A}} &= 0.5 \\
S_\nu &= 1\ \mathrm{mJy} \\
T_{\mathrm{sys}} &= 200\ \mathrm{K}
\end{aligned}$$

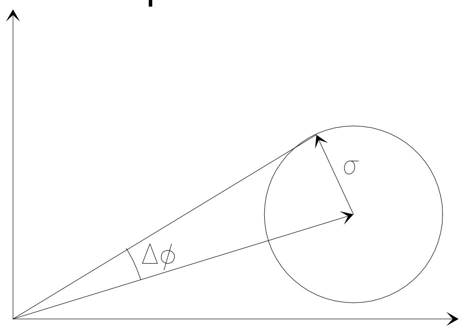
leads to a 3σ detection, and a total energy received from the source of $5\ 10^{-14}$ J.
(the received energy in 1 Million year would be about the kinetic energy of a falling snowflake...)

- Noise properties for 1 baseline vary with Signal-to-Noise ratio
- On the amplitude & flux density

$$S << \sigma \quad \begin{cases} \sigma_A & \simeq \sigma\sqrt{2-\frac{\pi}{2}}\left(1+\left(\frac{S}{2\sigma}\right)^2\right) \\ <S> & \simeq \sigma\sqrt{\frac{\pi}{2}}\left(1+\left(\frac{S}{2\sigma}\right)^2\right) \end{cases} \tag{40}$$

$$S >> \sigma \quad \begin{cases} \sigma_A & \simeq \sigma \\ <S> & \simeq S \end{cases} \tag{41}$$

- On the phase



$$S << \sigma \quad \left\{ \sigma_\phi \simeq \frac{\pi}{\sqrt{3}}\left(1-\sqrt{\frac{9}{2\pi^3}}\frac{S}{\sigma}\right) \right. \tag{42}$$

$$S >> \sigma \quad \left\{ \sigma_\phi \simeq \frac{\sigma}{S} \right. \tag{43}$$

- Source detection is much easier on the *phase* than on the *amplitude*, since for $S/N \sim 1$, $\sigma_\phi = 1\mathrm{radian} = 60^\circ$.

Conclusions

- mm interferometry is not so difficult to understand
- even if you don't, the noise equation is all you need
- the noise equation

$$\Delta T_{\rm b} = \frac{T_{\rm sys}}{\eta n \sqrt{\Delta\nu t}} \left(\frac{\theta_{\rm P}}{\theta_{\rm S}}\right)^2 \quad (44)$$

  allows you to check quickly if a source of given brightness $T_{\rm b}$ can be imaged at a given angular resolution $\theta_{\rm S}$ and spectral resolution $\Delta\nu$ (n is the number of antennas, $\theta_{\rm P}$ their primary beam width, and η an efficiency factor of order 0.5)
- $T_{\rm sys}$ is easy to guess: the simplistic value of 1 K per GHz of observing frequency is a good enough approximation in most cases.
- and **you know** $T_{\rm b}$ because you know the physics of your source!
- that is (almost) all you need to decide on the feasibility of an observation...