Seeing Constraints in Adaptive Optics Calibrations

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Using the seeing database from the Paranal observatory and a simple adaptive Optics (AO) model, we derive observing constraints and statistical errors on the PSF calibrations. We show that accurate PSF calibration (better than a few percent error on the Strehl ratio) can not be performed with the usual "nodding" approach (interlaced calibration).

1. Calibration requirements in adaptive optics

Current adaptive optics systems provide only partial compensation, that is, the images are not fully diffraction limited. Because of the limitation in the AO systems, both technological or fundamental, various errors limit the achieved Strehl ratio. Fitting, servolag and noise are the major limitations. Anisoplanatism can dominate when using off-axis guide stars, but we do not consider this problem here. The performance of an AO system is in the most general sense a function of the number of actuators per r_0 at the image wavelength and the number of photons per r_0^2 and per τ_0. It is well known that the performance, therefore the PSF, in term of Strehl ratio and shape, depends not only upon the wavelength of observation, but also of the actual seeing conditions during the observation (both being related by the above mentioned dependence in $r_0(\lambda_{\mathrm{im}})$).

Practically speaking, partially compensated PSFs have Strehl ratios anywhere between a few percent and close to 100%. The shape is usually bi-modal, with a nearly diffraction-limited core, broadened by tip–tilt residuals and blurred by fixed aberrations, plus a turbulence residual that shows as an extended halo. This is inconvenient for most of the astrophysical applications: It leads to ambiguities to whether what we see is due to these residual PSF features or real object features. To get rid of this ambiguity, one needs to *calibrate* the PSF. This calibration may later be used, through proper reduction procedures (deconvolution, PSF fitting) to extract accurate physical information from the AO compensated images/spectra.

This problem is known since the very beginning of adaptive optics. The most commonly used approach to calibration has been the following: if no PSF is readily available in the image within the isoplanatic patch around the science object –which represents the vast majority of the cases,– then one usually calibrates the PSF by observing a calibration star before and/or after the science exposure. This is adequate to a first approximation to calibrate the fixed aberrations, providing that they are not changing too drastically with telescope position. However, it is not at all accurate for the calibration of residual turbulent features. Indeed, years of seeing monitoring in various astronomical sites show that the seeing is not the *stationary* random process that was theorized, but that its average value fluctuates. These fluctuations will induce performance changes, and in turn PSF changes. *By Calibrating the PSF in a non-simultaneous manner, one therefore makes an error which is proportional to the seeing change between science observation and PSF calibration.*

To estimate the final PSF calibration errors, we need therefore to answer two questions: how much does the seeing vary with time, and how much will the PSF vary consequently ?

Another question to address is: which metric shall we use to measure PSF variations ? This is not a trivial question, as it strongly depends upon the method chosen for the processing of the data. This is too complex to be addressed in this paper. In order to simplify the problem, we have chosen the Strehl ratio as the PSF metric.

The problem can therefore be solved as follow: Using the Paranal seeing database, we first derive the statistics of the seeing variations versus time (e.g. the fractional seeing change). Using a simple AO model, we then express these seeing changes into Strehl ratio changes. The final goal is to compute the statistics of the Strehl ratio changes between two or more finite exposures.

2. Seeing data

We used the Paranal seeing database. The seeing is defined here as the Full Width at Half Maximum of the image at 500nm, at zenith. We also call this quantity ω in the following. Seeing data are collected at Paranal since 1989, with a Differential Image Motion Monitor (DIMM, **ref). The DIMM delivers an average r_0 value every two minute or so, 365 nights a year (obviously not during cloudy nights). The DIMM has been calibrated. Except for high winds, it is bias free and provides an accurate measurement of the seeing (the error was measured to be $\pm$ 10% at 0.25 arcsec seeing, therefore quite small for any practical purpose). A more complete description of the DIMM can be found in Sarazin and Roddier (1990). In the following, we used the 1994 data, not because they are any better than other years' data, but because the 1994 database is the most complete and corresponds to a period after the summit was flattened to its final shape and before any heavy work was started on the summit. The 1994 database contains 35000 seeing measurements.

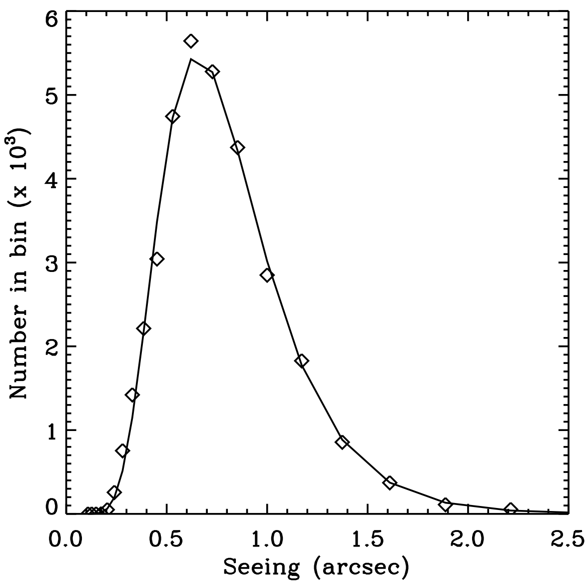
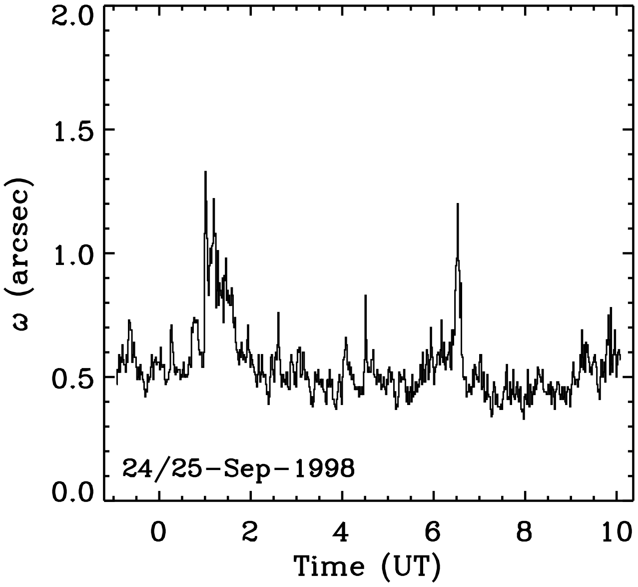


Fig. 1. Seeing histogram for the complete 1994 data set

Figure 1 presents the seeing histogram for the complete 1994 database. The median seeing value is 0.65", the average seeing is 0.70". A log normal distribution provides a satisfactory fit to the data (a least square fit to $\log_{10}$(seeing) gives the following gaussian parameters: average=-0.18, standard deviation=0.17). Note that 0.65" is also the overall median seeing value at Paranal, from 9 years of seeing monitoring. It is also interesting to note that the distribution found here has exactly the same standard deviation ($\sigma = 0.17$) than the one found by Racine for Mauna Kea (Racine 1996).

Figure 2 presents examples of typical nights of Paranal seeing, obtained recently with a 50s sampling time on the upgraded Paranal DIMM. The average seeing for the first night (24/25-09-98) is 0.54" and 0.75" for the second night (26/27-09-98). As can be seen, the first night is relatively quiet, with overall constant seeing, with some "bursts" of worse seeing. The second night displays variability at all time scales, up to several hours. In both cases though, there are rapid and sometimes large variations of seeing on time scale of minutes. For our problem, we are mostly interested in this short to mid term variations. One way to characterize these temporal seeing changes is the fractional seeing change, introduced by Racine (1996) and used by Sarazin (1997). The fractional seeing change (FSC) is a convenient quantity, easily understandable, which expresses the *relative* seeing change on a given time base. It is defined as:

$$FSC(\Delta t) = 2 \left\langle \frac{|\omega(t+\Delta t) - \omega(t)|}{|\omega(t+\Delta t) + \omega(t)|} \right\rangle_t \tag{1}$$



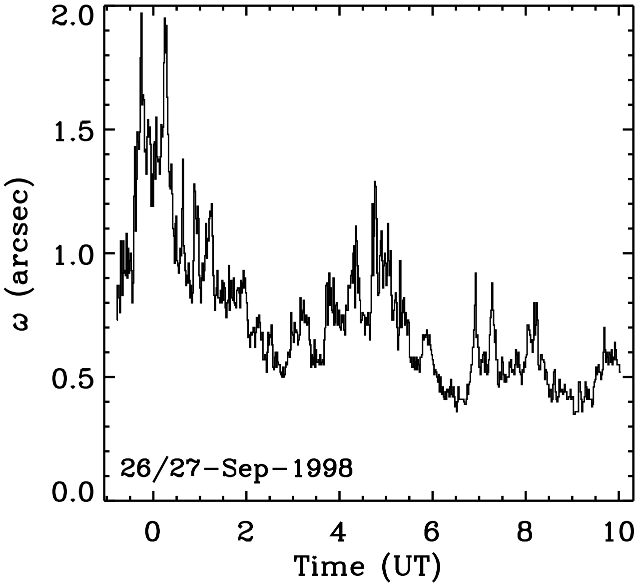


Fig. 2. Seeing versus time for Two typical nights at Paranal

The FSC for the 1994 Paranal data is plotted on Fig 3, left hand side, dashed line. It is similar to the one found by Racine for Mauna Kea data, with a steep raise at very short Δt and a smoother increase at larger time separation. The rate of change of the seeing is indeed very fast: for instance, for Δt of 10mn, the fractional seeing change is 0.13, which means that between two instantaneous[1] exposures separated by 10 minutes, the seeing will have changed, *in average*, by 13%. "Instantaneous" is the key word that does not fit our reality. In AO observations, exposure times are finite. The best one can do with the classical PSF calibration method is to switch to the calibrator right after the exposure on the science object is finished. For the sake of simplicity, let us assume that the exposure time on the PSF calibrator is the same than on the object (the conclusions are not affected even if this is not completely fulfilled). Then, an AO observation can be described as an exposure of length τ on the object, immediately followed (switch time is assumed negligible) by an exposure of the same duration on the PSF calibrator. Therefore, we are not interested immediately by the FSC as defined above, but by the fractional seeing change between two consecutive exposures of time $\tau = \Delta t$, that is the Finite Exposure Fractional Seeing Change:

$$FEFSC(\Delta t) = 2\left\langle \frac{|\omega(t+\Delta t, \tau=\Delta t) - \omega(t, \tau=\Delta t)|}{|\omega(t+\Delta t, \tau=\Delta t) + \omega(t, \tau=\Delta t)|} \right\rangle_t \tag{2}$$

where $\omega(t,\tau)$ is the *average* seeing, at time t, over a finite period of time τ. This function is plotted in the same Fig 3, left hand side, solid line. It does not raise as fast as the FSC, as the seeing fluctuations within the exposure are averaged out (by definition of the FEFSC). This leads for instance to a fractional change of 10% (instead of 13% for the FSC) for 10mn time lag.

Before estimating the effect of this seeing change on the PSF Strehl error, let us answer a last question: How does the seeing change behave between multiple consecutive exposures, in other words what is the change between the seeing during the PSF observation and the seeing during the object observation if we do not only do science object(SO)/PSF, but several series of these: SO/PSF/SO/PSF...

This was computed and is shown in the right hand side of Fig 3, for a SO/PSF sequence (upper curve), for a SO/PSF/SO/PSF sequence (middle curve) and for a 4×SO/PSF sequence (lower curve). What is computed is the fractional change between the *average* seeing on the object and the average seeing on the PSF. As could be expected, the error decreases as the square root of the number of exposures, and the fractional change for the 4×SO/PSF sequence is only 2 times lower than for a single SO/PSF sequence.

[1] In reality, remember that our seeing values are statistical average over approximately 100 seconds

Practically speaking, this means that the seeing change can be lowered in two ways: Either by reducing the exposure time (Δt), keeping the number of exposures constant, or by increasing the number of interlaced exposures SO/PSF, at constant exposure time. If the final goal, for instance is to keep the seeing change below 2%, this can be achieved in a single SO/PSF sequence with exposure time of approximately 80 seconds. For more realistic exposure time of 5 minutes, a sequence of $16\times$SO/PSF has to be performed.

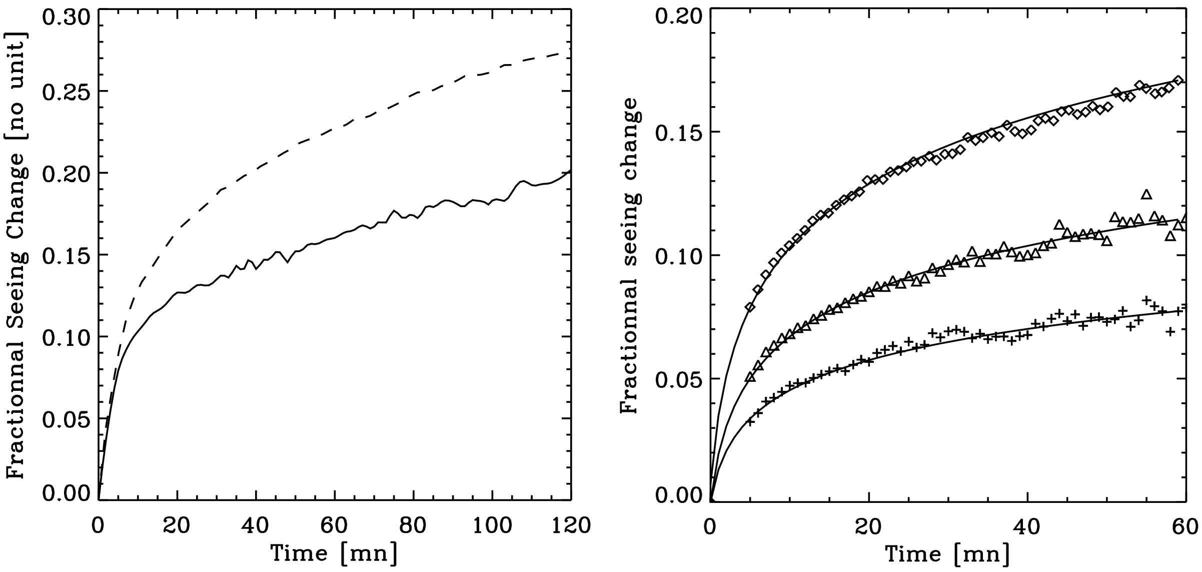


Fig. 3. Left: Fractional seeing change (dashed line) and finite exposure fractional seeing change (solid line). Right: Multiple finite exposures fractional seeing change. The upper curve is between two consecutive exposures, as in left hand figure. The curve below is between two interlaced sets of two exposures each (4 exposures in total) and the lower curve is between two interlaced sets of four exposures (8 exposures in total).

3. AO system model

At this point, we need a model of the AO system to derive the performance in term of Strehl versus r_0. Using this model, and the statistics of r_0 found in the previous section, we can deduce the statistics of the image Strehl ratio $\mathcal{S}$. The reasoning is the following: Let assume that we have the Strehl ratio versus r_0 relationship. Then, for any Strehl ratio value, we know how much the Strehl ratio will change for a given change in r_0 (read value of the FSC or FEFSC).

Several models could be used (e.g. Parenti & Sasiela, 1994). However, those models, because they want to reflect the reality, need quite a large number of input parameters. For the relationship between $\mathcal{S}$ and r_0 that we seek, we need at least an additionnal parameter, the telescope diameter (or D/r_0) that will affect the shape of the halo versus the central airy pattern, therefore that will affect the Strehl ratio. To keep the results of this paper more general, we have chosen a more simpler model (the simplest model?):

$$\mathcal{S} = exp(-\sigma_\varphi^2) \qquad \text{where} \qquad \sigma_\varphi^2 = \gamma r_0^{5/3} \tag{3}$$

This is a well known and often used approximation in which the Strehl ratio is equated to the coherent energy. This formulation is considered to be valid for $\mathcal{S} > 10\%$ approximately. γ is a constant, which value has no relevance here, as we are interested not in the absolute Strehl ratio versus r_0, but in the Strehl ratio *change*, for a given r_0 change, versus the Strehl ratio.

Equation 3 is plotted in figure 4, left hand side (diamond). The Strehl ratio change (SRC) for a r_0 relative change of 10% is also plotted (solid line). It was computed simply as:

$$SRC(r_0) = |\mathcal{S}(0.95r_0) - \mathcal{S}(1.05r_0)| \tag{4}$$

The normalized Strehl ratio change (dashed line) is the Strehl ratio change divided by the Strehl ratio.

Once these curves are obtained, it is straightforward to plot the (normalized) Strehl ratio change versus the Strehl ratio. This was done in Fig 4, right hand side, for a collection of $FEFSC$ values. In the figure, the normalized Strehl ratio change is labelled "Strehl error", as it can ultimately be viewed as the error on the estimated image Strehl ratio one does by using this non simultaneous PSF calibration.

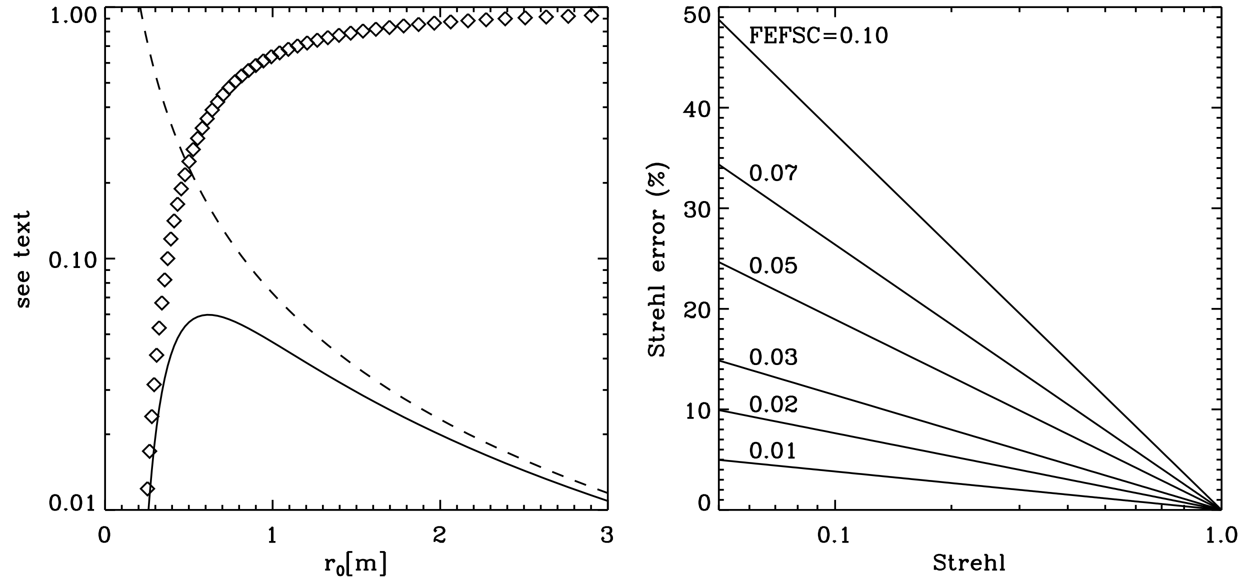


Fig. 4. Left: Strehl ratio (diamonds) versus r_0. Strehl change (solid line) and relative Strehl change (dashed line) for a FEFSC of 10%. Right: Relative Strehl ratio change versus Strehl ratio for various FEFSC value.

An example of how these curves can be read is as follow: Let us assume a FEFSC value of 0.1, corresponding to Δt of 10mn (see previous section). The error made on the Strehl of the PSF calibration, for an image strehl ratio of 30%, will be $\approx$ 20%. One can obviously use these curves in the opposite way, and determine, for a given Strehl ratio and a given error, the value of the FEFSC, therefore the maximum amount of time between the Science object (SO) and the PSF calibrator observations. At this point, it is worthwhile to remember that the FEFSC value decreases as the number of exposures in the serie SO/PSF/SO/PSF...

Let us now work out a numerical example. Assuming a Δt of 10mn, a goal of 2% Strehl error and a average Strehl ratio of 30%, how many round trips SO/PSF are needed ? As said above, the Strehl error is 20% for one round trip SO/PSF. A 2% Strehl error can be achieved for a FEFSC value of 0.01 (Fig 4). 100 round trips SO/PSF are needed to effectively reduce the overall seeing change down to 0.01 (FEFSC of 0.1 / square root of 100). This is much more than what is currently used in most of AO science observations (at most several round trips, most of the time just one or two), and quite unpractical.

4. Conclusion

Using seeing statistics from the Paranal seeing database, and a simple AO model, we have described a simple way to estimate the error made on the Strehl ratio of the PSF calibrator in AO observation. This Strehl error translates in effect into a photometry error, and may adversely affect detection of faint structure around the main object (if any).

We have shown that for regular observing parameters (a few minutes between object and PSF calibrator, Strehl ratio of 30%, several round trips between SO and PSF calibrator), the above mentionned Strehl error was very significant (5-20%). The only way to reduce it is either to increase the number of round trip, which is probably unadequate because extremely time consumming, or to reduce the Δt between SO and PSF exposures. This could be seen as a "chopping" procedure between SO and PSF. This would require modification of the Current AO system that are not designed to switch fast between two objects. It is also incompatible with observation of faint objects for which long exposure time are needed.

Another solution to this problem is to have a PSF calibration simultaneous with the Science object observation. This was proposed years ago and implemented on the CFHT PUEO curvature system (Véran et al 1997).

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